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Oceanic diffusion diagrams*

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Abstract—Some empirical relations between diffusion characteristics are investigated by the use of carefully examined data from instantaneous dye-release experiments in the upper mixed layer of the sea. The data cover a time scale of diffusion ranging from 2 hr to 1 month and a length scale from 30 m to 100 km. Two kinds of ' diffusion diagrams ' are prepared; one showing horizontal variance versus diffusion time and the other showing apparent diffusivity versus the scale of diffusion. The overall behaviors of the horizontal variance and of the apparent diffusivity are evidently different from those which the similarity theory of turbulence deduces. However, there still remains a possibility that the similarity theory may be valid locally with different values of the rate of turbulent energy dissipation. The diagrams provide a practical means to predict the rate of horizontal spread of substance from an instantaneous source.

INTRODUCTION

OCEANIC diffusion of contaminants is not only a matter of theoretical interest but is also a problem of practical importance in connection with water pollution. The process of oceanic diffusion is quite complex, and hardly any single theory can explain or interpret the entire pattern of diffusion. In addition to theoretical studies, therefore, progress in oceanic diffusion still depends heavily on an empirical approach by means of tracer experiments. Among various types of tracers used in these experimental studies, fluorescent dyes have proved the most promising; especially, rhodamine B is considered most accessible, stable, harmless, and convenient for use in the sea (CARPENTER, 1960; JAPANESE GOVERNMENT AGENCIES, 1958).

Since PRITCHARD and CARPENTER (1960) developed a new field technique for the direct continuous observation of the fluorescent concentration with the aid of a fluorometer, experimental data of oceanic diffusion with rhodamine dyes have been accumulated. An internationally cooperative experiment, Operation RHENO, in the North Sea made it possible to cover a scale of diffusion ranging up to 100 km and a time scale up to one month. Toward the other end of the scale, a set of experiments off the coast of Southern California provided the data for small scale diffusion ranging from 10 to 100 m. A fairly large amount of data is available in various sea areas for intermediate scales of diffusion ranging, say from 1 to 10 km.

Using the data of horizontal diffusion obtained prior to 1961, OKUBO (1962) presented a set of diagrams showing the empirical relationship between two parameters of diffusion. In this paper, we present a new set of diffusion diagrams on the basis of new data accumulated since 1961. One of the purposes of the present study is to compare the findings in the old diffusion diagrams with the new set of diagrams.

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DATA USED IN THE PRESENT STUDY

The author has found 20 sets of diffusion experiments that provide data suitable for the present purpose. The criteria for defining the suitability are: (i) The release of dye should be as close to a type of instantaneous point source as possible. The duration of release and the initial size of dye patch must be reported. At least the initial size must be estimated somehow to an order of magnitude. For a known initial size or more precisely a known initial variance, a critical time after which the diffusion is regarded practically as from a point source can be estimated from a theoretical relationship for the rate of change of variance. Thus the theory of JOSEPH and SENDNER (1958) gives

$$\sigma_0^2 = 6\,P^2 t_0^2 \tag{1}$$

where σ_0^2 is the variance associated with the initial distribution of patch; P is a diffusion velocity* being taken 1 cm sec^{-1} as a representative value after Joseph and Sendner, and t_0 is a characteristic time of diffusion during which a patch from a point source grows into the size, σ_0. Obviously the critical time must be much larger than t_0. We choose 10 t_0 as the critical time. Diffusion data prior to the critical time are disregarded for the present purpose. (ii) The dye patch should maintain a sufficient distance from vertical boundaries so that the field of diffusion is regarded as extending to infinity in the horizontal direction. In most cases whether a dye patch has felt boundaries or not can be determined intuitively from the observed pattern of concentration distribution. The presence or absence of constraints in the vertical direction, e.g. the bottom, is expected to produce little change in the horizontal variance but rather to have a significant influence in the time behaviour of the maximum concentration of patch. (iii) The horizontal distribution of dye concentration should be observed in such a manner that the variance can be computed directly from the distribution. Aerial photographs of the dye patch are not considered useful for this purpose, although some progress has been made in determining the concentration distribution from dye photos (ICHIYE and PLUTCHAK, 1966). (iv) The observations should cover the greater part of a dye patch so that the estimated amount of dye remaining within the patch is comparable to the total amount of dye released, when the loss of dye due to photochemical decay, physical adsorption, etc. is taken into account if possible. PRITCHARD and CARPENTER (1960) in a diffusion study in Chesapeake Bay, reported a mass balance of 70–80%. This case is ideal, however. Usually mass balances as low as 50% may be taken as permissible especially in the later stage of diffusion. Quite often the mass balance is neither reported nor computable simply because of lack of information on the vertical distribution of dye. In this case, we estimate an appropriate value of the layer within which dye is distributed nearly uniformly. The amount of dye in the patch is then computed from the horizontal distribution of dye multiplied by the thickness of the layer.

COMPUTATION OF DIFFUSION CHARACTERISTICS

An oceanic diffusion diagram is defined as a plot of a characteristic parameter of diffusion against another such parameter. A set of fundamental parameters is the second central moment of the dye distribution, i.e. the variance and 'diffusion time', i.e. the time elapsed since the introduction of tracer. The variance of the

*KULLENBERG (1969) obtains an interesting relationship between P and the initial *vertical* extent of dye.

horizontal distribution is certainly a suitable measure of the spread of the substance. On the other hand, the maximum concentration in a patch of dye may not be a good measure of diffusion, not only because the observation of the maximum concentration involves a great deal of uncertainty but also because the peak concentration is very sensitive to the decay of dye due to physico-chemical processes if appreciable. So long as the decay of dye obeys the law of the first order kinetics, the value of the variance computed from the data uncorrected for decay would be the same as that from corrected data. In this respect, the variance is one of the most stable parameters of diffusion.

Since the horizontal dye-distribution usually appears asymmetrical in the sense that a characteristic length is larger in one direction than another [see an example in a paper by JOSEPH, SENDNER and WEIDEMANN (1964)], two variances, say along the major and minor axes of the dye patch, are necessary to properly describe ocean diffusion. However, because of the extremely irregular pattern of dye distribution, we conveniently measure the area enclosed by an isoconcentration curve and then define the radius of a circle of equal area (JOSEPH and SENDNER, 1958). A radially symmetrical distribution $S(t,r_e)$, thus characterized by the equivalent radius, r_e, and the diffusion time, t, is the basic distribution from which the horizontal variance, i.e. the mean square distances from the centre of mass may be computed as

$$\sigma_{rc}^2 \equiv \int_0^\infty r_e^2 S \, 2\pi r_e \mathrm{d}r_e \bigg/ \int_0^\infty S \, 2\pi r_e \mathrm{d}r_e \quad . \tag{2}$$

Note that the variance σ^2_{rc} thus defined on the radially symmetrical distribution is twice the mean variance which is ordinarily used for a two-dimensional distribution.

The accuracy of the estimate of σ^2_{rc} depends primarily on uncertainties in the equivalent radii. ‘ Good ’ data by which is meant that a fairly precise distribution of dye concentration is observed, should provide an estimate of σ^2_{rc} within a precision of 5%. Completely independent evaluations of the equivalent radii by the use of the same raw data reveal that the relative difference in r^2_e is a few per cent.

The early stages of diffusion are difficult to sample since the patch has small spatial extent and any significant disturbance to the patch by a sampling vessel must be avoided. With a view toward minimizing the disturbances due to a vessel yet determining the variances of the concentration distribution, FOXWORTHY, TIBBY and BARSOM (1966) carried out dye experiments in which a small sampling vessel made single crossings of dye patches of small size down to 30 m. The crossings were aimed so as to lie along the direction of elongation of a patch and the transverse direction so that the variances along the principal axes of the concentration distribution could be calculated. The observed dye-distributions have been fitted to a Gaussian curve to compute the variances. The fit was found to be reasonably good according to Foxworthy, Tibby and Barsom.

Since the two crossings along the principal axes were not carried out simultaneously, an empirical relation between the estimated variance and diffusion time was determined for each axis. It can be shown that, if σ^2_X and σ^2_Y denote the variances in the major and minor principal axes, respectively, and if the distribution is Gaussian, then the variance σ^2_{rc} for a radially symmetrical distribution obtained by taking the equivalent radius is given by

$$\sigma^2_{rc} = 2\,\sigma_X \sigma_Y \quad .$$

All other dye experiments here quoted consist of measuring the horizontal concentration distribution by a proper network of crossings in a dye patch.

We also compute an apparent diffusivity K_a which is defined by $\sigma^2_{rc}/4t$. The reason why we choose this definition is because, for a two-dimensional Fickian diffusion, K_a becomes K, a constant diffusivity. The results of computations of these diffusion parameters are summarized in Table 1.

DIFFUSION DIAGRAMS

By plotting the value of the variance, σ^2_{rc}, against diffusion time, t, we obtain a basic diffusion diagram (Fig. 1). The variance ranges from 10^7 cm^2 to 10^{13} cm^2, while

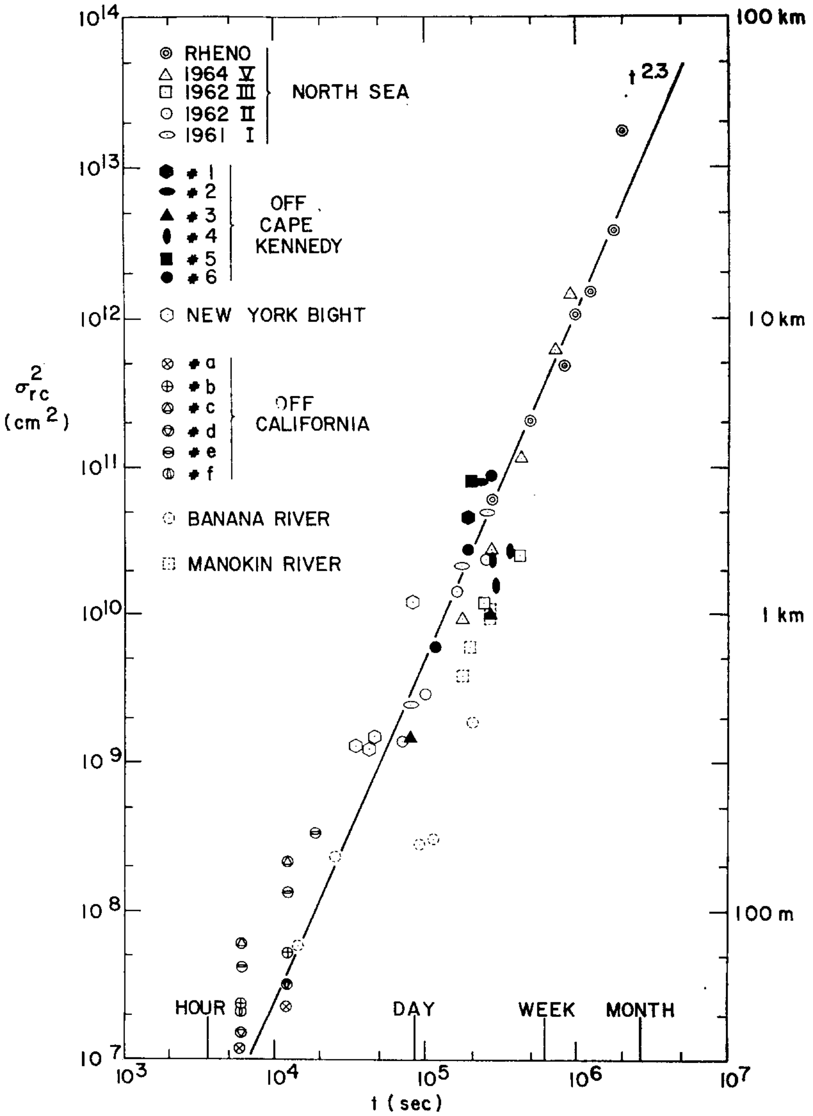

Fig. 1. A diffusion diagram for variance versus diffusion time.

Table 1. Computation of diffusion characteristics.

Experiment	t(sec)	σ_{rc}^{2}(cm^2)	σ_{rc}(cm)	K_a(cm^2/sec)	l(cm) $\equiv 3\sigma_{rc}$	ρ	σ^{2}_{r}(cm^2)	*Method of computation of* σ_{rc}^{2}	*Source of data*
' RHENO ' (1956) North Sea (Aug.-Sept.)	$2\cdot7 \times 10^{5}$	$6\cdot1 \times 10^{10}$	$2\cdot47 \times 10^{5}$	$5\cdot65 \times 10^{4}$	$7\cdot41 \times 10^{5}$	NA	NA	GD	Weidemann and Sendner (Private communication)
	$4\cdot86 \times 10^{5}$	$2\cdot05 \times 10^{11}$	$4\cdot53 \times 10^{5}$	$1\cdot05 \times 10^{5}$	$1\cdot36 \times 10^{6}$	NA	NA	GD	
	$8\cdot1 \times 10^{5}$	$4\cdot8 \times 10^{11}$	$6\cdot93 \times 10^{5}$	$1\cdot48 \times 10^{5}$	$2\cdot08 \times 10^{6}$	NA	NA	DC	
	$9\cdot73 \times 10^{5}$	$1\cdot08 \times 10^{12}$	$1\cdot04 \times 10^{6}$	$2\cdot78 \times 10^{5}$	$3\cdot12 \times 10^{6}$	NA	NA	DC	
	$1\cdot22 \times 10^{6}$	$1\cdot50 \times 10^{12}$	$1\cdot22 \times 10^{6}$	$3\cdot08 \times 10^{5}$	$3\cdot66 \times 10^{6}$	NA	NA	DC	
	$1\cdot8 \times 10^{6}$	$3\cdot8 \times 10^{12}$	$1\cdot95 \times 10^{6}$	$5\cdot2 \times 10^{5}$	$5\cdot85 \times 10^{6}$	NA	NA	DC	
	$2\cdot01 \times 10^{6}$	$1\cdot75 \times 10^{13}$	$4\cdot17 \times 10^{6}$	$2\cdot2 \times 10^{6}$	$1\cdot24 \times 10^{7}$	NA	NA	GE	
1964—V North Sea	$1\cdot69 \times 10^{5}$	$1\cdot0 \times 10^{10}$	$1\cdot0 \times 10^{5}$	$1\cdot48 \times 10^{4}$	$3\cdot0 \times 10^{5}$	NA	NA	DC	Weidemann and Sendner (Private communication)
	$2\cdot63 \times 10^{5}$	$2\cdot9 \times 10^{10}$	$1\cdot70 \times 10^{5}$	$2\cdot76 \times 10^{4}$	$5\cdot1 \times 10^{5}$	NA	NA	GD	
	$4\cdot14 \times 10^{5}$	$1\cdot18 \times 10^{11}$	$3\cdot44 \times 10^{5}$	$7\cdot2 \times 10^{4}$	$1\cdot03 \times 10^{6}$	NA	NA	GD	
	$7\cdot02 \times 10^{5}$	$6\cdot3 \times 10^{11}$	$7\cdot95 \times 10^{5}$	$2\cdot2 \times 10^{5}$	$2\cdot38 \times 10^{6}$	NA	NA	GE	
	$9\cdot0 \times 10^{5}$	$1\cdot48 \times 10^{12}$	$1\cdot2 \times 10^{6}$	$4\cdot1 \times 10^{5}$	$3\cdot6 \times 10^{6}$	NA	NA	GE	
Oct. 1962—III North Sea	$2\cdot38 \times 10^{5}$	$1\cdot27 \times 10^{10}$	$1\cdot13 \times 10^{5}$	$1\cdot34 \times 10^{4}$	$3\cdot39 \times 10^{5}$	0·25	$2\cdot73 \times 10^{10}$	GD	Joseph, Sendner and Weidemann (1964)
	$4\cdot1 \times 10^{5}$	$2\cdot65 \times 10^{10}$	$1\cdot63 \times 10^{5}$	$1\cdot61 \times 10^{4}$	$4\cdot90 \times 10^{5}$	0·18	$7\cdot7 \times 10^{10}$	GD	
Sept. 1962—II North Sea	$6\cdot8 \times 10^{4}$	$1\cdot5 \times 10^{9}$	$3\cdot9 \times 10^{4}$	$5\cdot5 \times 10^{3}$	$1\cdot17 \times 10^{5}$	0·33	$2\cdot48 \times 10^{9}$	GD	Joseph, Sendner and Weidemann (1964)
	$9\cdot65 \times 10^{4}$	$3\cdot1 \times 10^{9}$	$5\cdot6 \times 10^{4}$	$8\cdot1 \times 10^{3}$	$1\cdot7 \times 10^{5}$	0·30	$5\cdot58 \times 10^{9}$	GD	
	$1\cdot59 \times 10^{5}$	$1\cdot5 \times 10^{10}$	$1\cdot2 \times 10^{5}$	$2\cdot36 \times 10^{4}$	$3\cdot7 \times 10^{5}$	0·33	$2\cdot48 \times 10^{10}$	GD	
	$2\cdot47 \times 10^{5}$	$2\cdot4 \times 10^{10}$	$1\cdot55 \times 10^{5}$	$2\cdot42 \times 10^{4}$	$4\cdot65 \times 10^{5}$	0·10	$1\cdot2 \times 10^{11}$	GD	
Nov. 1961—I North Sea	$7\cdot6 \times 10^{4}$	$2\cdot7 \times 10^{9}$	$5\cdot2 \times 10^{4}$	$8\cdot9 \times 10^{3}$	$1\cdot56 \times 10^{5}$	0·33	$4\cdot45 \times 10^{9}$	DC	Joseph, Sendner and Weidemann (1964)
	$1\cdot73 \times 10^{5}$	$1\cdot1 \times 10^{10}$	$1\cdot05 \times 10^{5}$	$1\cdot58 \times 10^{4}$	$3\cdot15 \times 10^{5}$	0·28	$2\cdot09 \times 10^{10}$	DC	
	$2\cdot49 \times 10^{5}$	$5\cdot1 \times 10^{10}$	$2\cdot26 \times 10^{5}$	$5\cdot1 \times 10^{4}$	$6\cdot78 \times 10^{5}$	0·20	$1\cdot33 \times 10^{11}$	GD	
Mar. 1962 No. 1 off Cape Kennedy	$1\cdot85 \times 10^{5}$	$4\cdot77 \times 10^{10}$	$2\cdot18 \times 10^{4}$	$6\cdot5 \times 10^{4}$	$6\cdot54 \times 10^{5}$	0·51	$5\cdot92 \times 10^{10}$	GD	Carter and Okubo (1965)
Mar. 1962 No. 2 off Cape Kennedy	$1\cdot66 \times 10^{5}$	$2\cdot22 \times 10^{10}$	$1\cdot49 \times 10^{5}$	$3\cdot3 \times 10^{4}$	$4\cdot47 \times 10^{5}$	0·19	$6\cdot11 \times 10^{10}$	GD	Carter and Okubo (1965)
	$2\cdot29 \times 10^{5}$	$8\cdot4 \times 10^{10}$	$2\cdot9 \times 10^{5}$	$9\cdot2 \times 10^{4}$	$8\cdot7 \times 10^{6}$	0·32	$1\cdot43 \times 10^{11}$	GD	

Table 1. *Continued.*

Apr. 1962 No. 3 off Cape Kennedy	$7{\cdot}6 \times 10^{4}$	$1{\cdot}58 \times 10^{9}$	$4{\cdot}0 \times 10^{4}$	$5{\cdot}2 \times 10^{3}$	$1{\cdot}2 \times 10^{5}$	0·36	$2{\cdot}45 \times 10^{9}$	GD	CARTER and OKUBO (1965)
	$2{\cdot}56 \times 10^{5}$	$1{\cdot}1 \times 10^{10}$	$1{\cdot}05 \times 10^{5}$	$1{\cdot}08 \times 10^{4}$	$3{\cdot}15 \times 10^{5}$	0·22	$2{\cdot}64 \times 10^{10}$	GD	
Aug. 1962 No. 4 off Cape Kennedy	$2{\cdot}69 \times 10^{5}$	$2{\cdot}42 \times 10^{10}$	$1{\cdot}55 \times 10^{5}$	$2{\cdot}3 \times 10^{4}$	$4{\cdot}65 \times 10^{5}$	NA	NA	GD	CARTER and OKUBO (1965)
	$2{\cdot}84 \times 10^{5}$	$1{\cdot}65 \times 10^{10}$	$1{\cdot}29 \times 10^{5}$	$1{\cdot}46 \times 10^{4}$	$3{\cdot}87 \times 10^{5}$	0·4	$2{\cdot}39 \times 10^{10}$	GD	
	$3{\cdot}56 \times 10^{5}$	$2{\cdot}82 \times 10^{10}$	$1{\cdot}68 \times 10^{5}$	$1{\cdot}98 \times 10^{4}$	$5{\cdot}04 \times 10^{5}$	0·19	$7{\cdot}76 \times 10^{10}$	GD	
Aug. 1962 No. 5 off Cape Kennedy	$1{\cdot}97 \times 10^{5}$	$8{\cdot}2 \times 10^{10}$	$2{\cdot}86 \times 10^{5}$	$1{\cdot}04 \times 10^{5}$	$8{\cdot}58 \times 10^{5}$	0·3	$1{\cdot}47 \times 10^{11}$	GD	CARTER and OKUBO (1965)
Aug. 1962 No. 6 off Cape Kennedy	$1{\cdot}12 \times 10^{5}$	$6{\cdot}3 \times 10^{9}$	$8{\cdot}0 \times 10^{4}$	$1{\cdot}42 \times 10^{4}$	$2{\cdot}4 \times 10^{5}$	0·14	$2{\cdot}33 \times 10^{10}$	GD	CARTER and OKUBO (1965)
	$1{\cdot}85 \times 10^{5}$	$2{\cdot}8 \times 10^{10}$	$1{\cdot}67 \times 10^{5}$	$3{\cdot}8 \times 10^{4}$	$5{\cdot}01 \times 10^{5}$	0·13	$1{\cdot}10 \times 10^{11}$	GD	
	$2{\cdot}66 \times 10^{5}$	$8{\cdot}95 \times 10^{10}$	$2{\cdot}99 \times 10^{5}$	$8{\cdot}4 \times 10^{4}$	$8{\cdot}97 \times 10^{5}$	0·16	$2{\cdot}91 \times 10^{11}$	GD	
Apr. 1964 Banana River	$1{\cdot}4 \times 10^{4}$	$6{\cdot}25 \times 10^{7}$	$7{\cdot}93 \times 10^{3}$	$1{\cdot}12 \times 10^{3}$	$2{\cdot}38 \times 10^{4}$	0·8	$6{\cdot}4 \times 10^{7}$	GD	CARTER and OKUBO (1965)
	$2{\cdot}4 \times 10^{4}$	$2{\cdot}62 \times 10^{8}$	$1{\cdot}62 \times 10^{4}$	$2{\cdot}75 \times 10^{3}$	$4{\cdot}86 \times 10^{4}$	0·5	$3{\cdot}28 \times 10^{8}$	GD	
	$8{\cdot}8 \times 10^{4}$	$3{\cdot}1 \times 10^{8}$	$1{\cdot}76 \times 10^{4}$	$8{\cdot}8 \times 10^{2}$	$5{\cdot}28 \times 10^{4}$	0·3	$5{\cdot}62 \times 10^{8}$	GD	
	$1{\cdot}1 \times 10^{5}$	$3{\cdot}35 \times 10^{8}$	$1{\cdot}83 \times 10^{4}$	$7{\cdot}7 \times 10^{2}$	$5{\cdot}5 \times 10^{4}$	0·26	$6{\cdot}87 \times 10^{8}$	GD	
	$1{\cdot}97 \times 10^{5}$	$2{\cdot}01 \times 10^{9}$	$4{\cdot}48 \times 10^{4}$	$2{\cdot}6 \times 10^{3}$	$1{\cdot}34 \times 10^{5}$	0·13	$8{\cdot}04 \times 10^{9}$	GD	
June 1967 Manokin River	$1{\cdot}7 \times 10^{5}$	$4{\cdot}25 \times 10^{9}$	$6{\cdot}5 \times 10^{4}$					GE	CARTER (1967)
	$1{\cdot}9 \times 10^{5}$	$6{\cdot}58 \times 10^{9}$	$8{\cdot}1 \times 10^{4}$					GE	
	$2{\cdot}6 \times 10^{5}$	$1{\cdot}16 \times 10^{10}$	$1{\cdot}08 \times 10^{5}$					GE	
Oct. 1961 New York Bight	$3{\cdot}38 \times 10^{4}$	$1{\cdot}42 \times 10^{9}$	$3{\cdot}77 \times 10^{4}$	$1{\cdot}05 \times 10^{4}$	$1{\cdot}13 \times 10^{5}$	01·4	$5{\cdot}25 \times 10^{9}$	GD	COSTIN, DAVIS, GERARD and KATZ (1963)
	$4{\cdot}13 \times 10^{4}$	$1{\cdot}38 \times 10^{9}$	$3{\cdot}72 \times 10^{4}$	$8{\cdot}4 \times 10^{3}$	$1{\cdot}11 \times 10^{5}$	NA	NA	GD	
	$4{\cdot}45 \times 10^{4}$	$1{\cdot}65 \times 10^{9}$	$4{\cdot}06 \times 10^{4}$	$9{\cdot}3 \times 10^{3}$	$1{\cdot}22 \times 10^{5}$	0·13	$6{\cdot}52 \times 10^{9}$	GD	
	$7{\cdot}9 \times 10^{4}$	$1{\cdot}29 \times 10^{10}$	$1{\cdot}13 \times 10^{5}$	$4{\cdot}1 \times 10^{4}$	$3{\cdot}39 \times 10^{5}$	NA	NA	GD	
	$9{\cdot}13 \times 10^{4}$	$1{\cdot}84 \times 10^{10}$	$1{\cdot}35 \times 10^{5}$	$5{\cdot}04 \times 10^{4}$	$4{\cdot}05 \times 10^{5}$	0·25	$3{\cdot}95 \times 10^{10}$	GE	
Oct. 1962 No. (a) off Southern California	$6{\cdot}0 \times 10^{3}$	$1{\cdot}23 \times 10^{7}$	$3{\cdot}51 \times 10^{3}$	$5{\cdot}13 \times 10^{2}$	$1{\cdot}05 \times 10^{4}$	0·17	$3{\cdot}64 \times 10^{7}$	GD	FOXWORTHY, TIBBY and BARSOM (1966)
	$1{\cdot}2 \times 10^{4}$	$2{\cdot}42 \times 10^{7}$	$4{\cdot}92 \times 10^{3}$	$5{\cdot}04 \times 10^{2}$	$1{\cdot}48 \times 10^{4}$	0·16	$7{\cdot}63 \times 10^{7}$	GD	

Table 1. *Continued.*

Nov. 1962 No. (b) off Southern California	$6\cdot0 \times 10^3$ $1\cdot2 \times 10^4$	$2\cdot58 \times 10^7$ $5\cdot57 \times 10^7$	$5\cdot08 \times 10^3$ $7\cdot46 \times 10^3$	$1\cdot07 \times 10^3$ $1\cdot16 \times 10^3$	$1\cdot52 \times 10^4$ $2\cdot26 \times 10^4$	0·40 0·32	$3\cdot8 \times 10^7$ $9\cdot64 \times 10^7$	GD GD	FOXWORTHY, TIBBY and BARSOM (1966)
Mar. 1963 No. (c) off Southern California	$6\cdot0 \times 10^3$ $1\cdot2 \times 10^4$	$6\cdot44 \times 10^7$ $2\cdot31 \times 10^8$	$8\cdot02 \times 10^3$ $1\cdot52 \times 10^4$	$2\cdot68 \times 10^3$ $4\cdot81 \times 10^3$	$2\cdot41 \times 10^4$ $4\cdot56 \times 10^4$	0·27 0·25	$1\cdot27 \times 10^8$ $4\cdot76 \times 10^8$	GD GD	FOXWORTHY, TIBBY and BARSOM (1966)
Apr. 1963 No. (d) off Southern California	$6\cdot0 \times 10^3$ $1\cdot2 \times 10^4$	$1\cdot60 \times 10^7$ $3\cdot46 \times 10^7$	$4\cdot00 \times 10^3$ $5\cdot88 \times 10^3$	$6\cdot67 \times 10^2$ $7\cdot21 \times 10^2$	$1\cdot2 \times 10^4$ $1\cdot76 \times 10^4$	0·29 0·21	$5\cdot04 \times 10^7$ $8\cdot78 \times 10^7$	GD GD	FOXWORTH, TIBBY and BARSOM (1966)
Aug. 1963 No. (e) off Southern California	$6\cdot0 \times 10^3$ $1\cdot2 \times 10^4$ $1\cdot8 \times 10^4$	$4\cdot56 \times 10^7$ $1\cdot41 \times 10^8$ $3\cdot6 \times 10^8$	$6\cdot75 \times 10^3$ $1\cdot19 \times 10^4$ $1\cdot90 \times 10^4$	$1\cdot9 \times 10^3$ $2\cdot95 \times 10^3$ $5\cdot0 \times 10^3$	$2\cdot02 \times 10^4$ $3\cdot57 \times 10^4$ $5\cdot70 \times 10^4$	0·26 0·21 0·18	$9\cdot22 \times 10^7$ $3\cdot49 \times 10^8$ $7\cdot66 \times 10^8$	GD GD GD	FOXWORTHY, TIBBY and BARSOM (1966)
Dec. 1963 No. (f) off Southern California	$6\cdot0 \times 10^3$ $1\cdot2 \times 10^4$	$2\cdot25 \times 10^7$ $5\cdot58 \times 10^7$	$4\cdot74 \times 10^3$ $7\cdot47 \times 10^3$	$9\cdot37 \times 10^2$ $1\cdot16 \times 10^3$	$1\cdot42 \times 10^4$ $2\cdot24 \times 10^4$	0·40 0·30	$3\cdot28 \times 10^7$ $1\cdot01 \times 10^8$	GD GD	FOXWORTHY, TIBBY and BARSOM (1966)

t, diffusion time; σ_{rc}^2, horizontal variance for radially symmetrical distribution; $K_a \equiv \frac{1}{4}\,\sigma^2_{rc}/t$, apparent diffusivity; $l \equiv 3\,\sigma_{rc}$, scale of diffusion; $\rho \equiv \sigma_y/\sigma_x$, degree of elongation; σ^2_r, horizontal variance for nonsymmetric distribution; NA, not available; (method of computation of σ_{rc}^2) GD fit to Gaussian distribution; GE, fit to generalized exponential distribution; DC, direct computation.

the time of diffusion ranges from 2 hours to nearly 1 month. Though the points in the diagram scatter somewhat, we can recognize an obvious trend that the variance grows much faster, in the power of t, than a linear increase with time; Fickian diffusion with a constant diffusivity would result in the variance growing linearly with time. A straight line fit by eye to all the data points gives the relationship that

$$\sigma^2_{rc} = 0{\cdot}0108\ t^{2{\cdot}34} \tag{3}$$

where σ^2_{rc} and t are expressed in terms of cm^2 and sec, respectively. A similar diagram based on data obtained prior to 1961 by a variety of techniques shows nearly the same behaviour (OKUBO, 1962, 1968). Although the quality of the old diffusion data is poor in comparison with that of the new data used in this paper, the general conclusion remains unchanged. Irrespective of the various macroscopic oceanographic conditions, the variance increases with time at a power between 2 and 3, indicating non-Fickian diffusion. The similarity theory of turbulence (KOLMOGOROV, 1941) predicts the third power of t for the variance of particle displacements initiated from an infinitesimally small source (BATCHELOR, 1950).

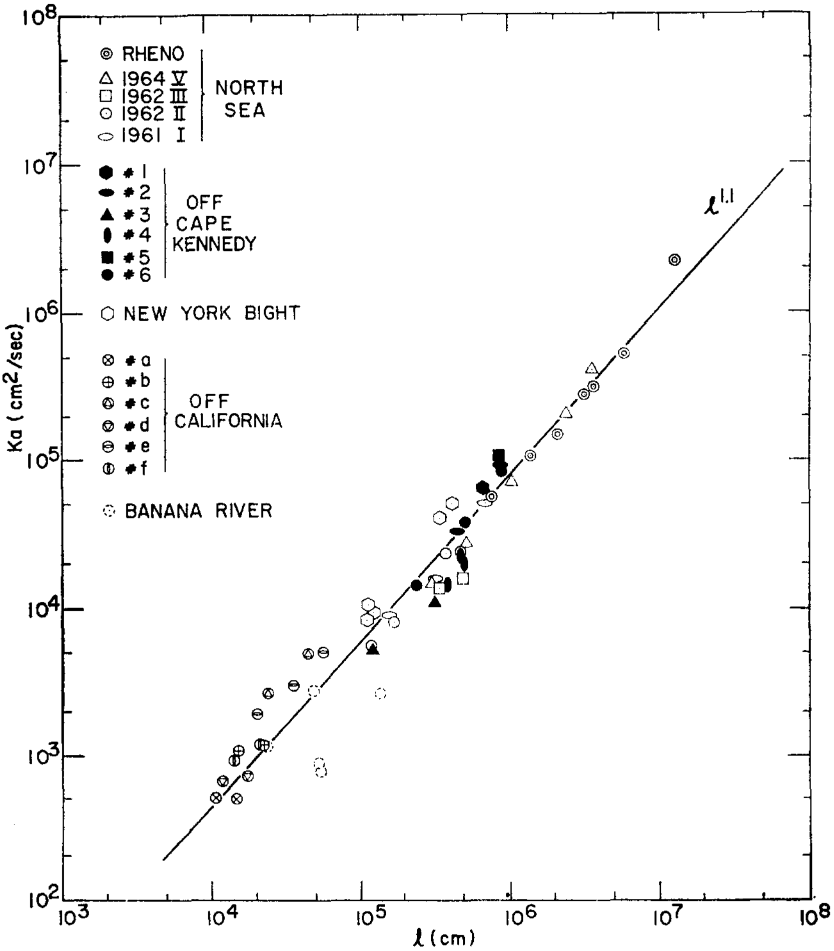

Fig. 2. A diffusion diagram for apparent diffusivity versus scale of diffusion.

Figure 2 shows the relationship between the apparent diffusivity, K_a, and a scale of diffusion, l. The scale of diffusion is arbitrarily defined as 3 σ_{rc}. If the radially symmetric distribution is Gaussian, 95 % of dye remains within the diameter of 3 σ_{rc}. A straight line fit by eye to the data points gives the empirical relationship that

$$K_a = 0{\cdot}0103\ l^{1{\cdot}15} \tag{4}$$

where K_a and l are expressed in terms of cm^2 sec^{-1} and cm, respectively, the exponent of l is apparently smaller than 4/3. Several investigators have proposed the 4/3 law of oceanic diffusion (STOMMEL, 1949; ICHIYE and OLSON, 1960, among others).

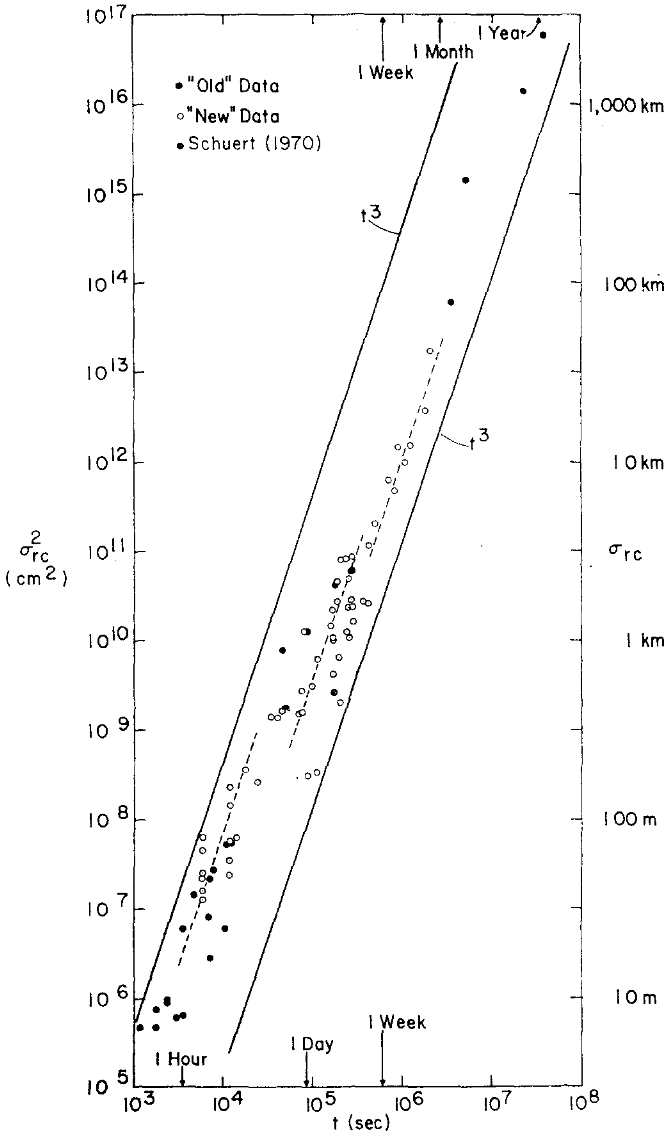

Fig. 3. Variance versus diffusion time (old and new data): fit of the third power law locally.

The present findings do not rule out the possibility that the third power law of the variance and equivalently the 4/3 power law of the diffusivity may be valid locally for some time and length scales. All that is required is that the rate of turbulent energy dissipation, ϵ (cm^2 sce^{-3}), varies with the time and length scales in which one is interested. To show this possibility, we plot all the data, old and new together, in one diagram for σ^2_{rc} versus t (Fig. 3) and for K_a versus l (Fig. 4). The third power relation and the 4/3 power relation are fitted locally by eye to the data points.

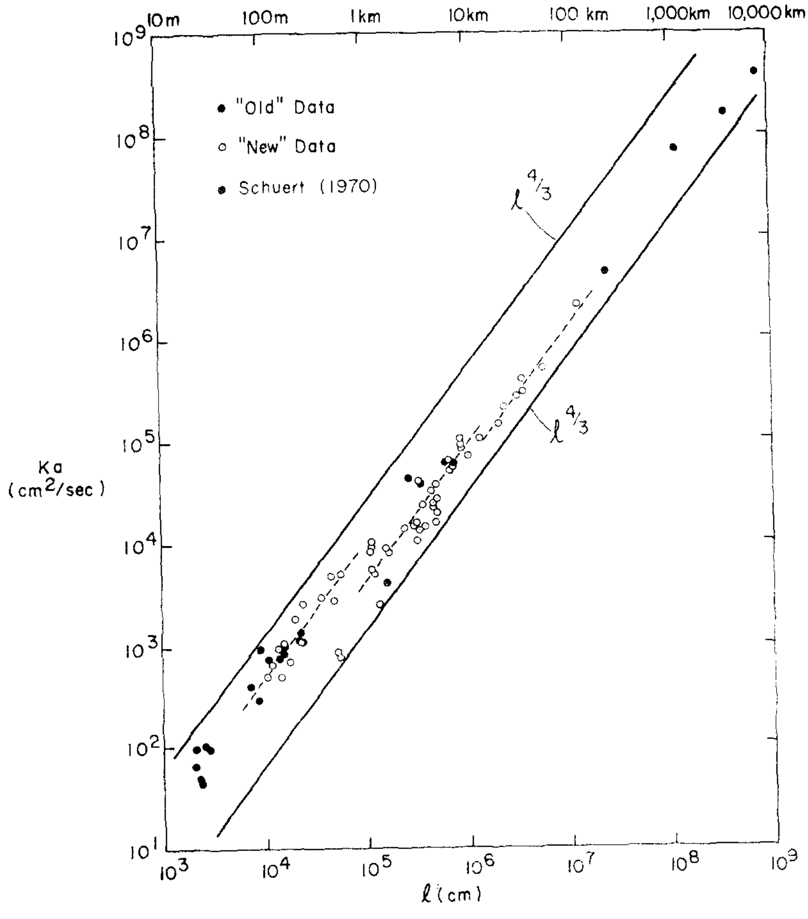

Fig. 4. Apparant diffusivity versus scale of diffusion (old and new data): fit of the 4/3 power law locally.

The new diffusion diagrams here constructed using the most reliable data from dye experiments provide not only a practical means of predicting the characteristics of oceanic diffusion but also a certain clue to the structure of oceanic turbulence. For the prediction of horizontal spread of substance from an instantaneous, small source, Figs. 1 and 2 can be used as a general guide. An example of predictions is given in Table 2. However, a few things must be remembered. First, the variance varies depending presumably upon the oceanographical conditions, by an order of magnitude or more for the same diffusion time. Secondly, the real diffusion pattern never shows radial symmetry. The variance σ^2_r for a nonsymmetric patch always exceeds the variance $\sigma^2{}_{re}$ given here. A correction can be made if the degree of elongation is known (see Table 1). Thirdly, the definition of the scale of diffusion is quite arbitrary. Consequently some care must be exercised in using the diagrams for predicting purposes.

Table 2. An example of predictions of diffusion characteristics.

(*a*) *Variance and standard deviation of a radially symmetric patch versus diffusion time.*

t(sec)		$\sigma_{rc}{}^2$(cm^2)	σ_{rc}(cm)
$3{\cdot}6 \times 10^3$	(1 hr)	$2{\cdot}3 \times 10^6$	$1{\cdot}5 \times 10^3$
10^4		$2{\cdot}5 \times 10^7$	$5{\cdot}0 \times 10^3$
$3{\cdot}6 \times 10^4$	(10 hr)	$4{\cdot}8 \times 10^8$	$2{\cdot}2 \times 10^4$
$8{\cdot}64 \times 10^4$	(1 day)	$3{\cdot}6 \times 10^9$	$6{\cdot}0 \times 10^4$
10^5		$5{\cdot}4 \times 10^9$	$7{\cdot}4 \times 10^4$
$6{\cdot}05 \times 10^5$	(1 week)	$3{\cdot}6 \times 10^{11}$	$6{\cdot}0 \times 10^5$
10^6		$1{\cdot}2 \times 10^{12}$	$1{\cdot}1 \times 10^6$
$2{\cdot}59 \times 10^6$	(1 month)	$1{\cdot}2 \times 10^{13}$	$3{\cdot}3 \times 10^6$

(*b*) *Apparent diffusivity versus scale of diffusion.*

l(cm)	K_a(cm^2/sec)
10^4	$4{\cdot}1 \times 10^2$
5×10^4	$2{\cdot}6 \times 10^3$
10^5	$5{\cdot}8 \times 10^3$
5×10^5	$3{\cdot}7 \times 10^4$
10^6	$8{\cdot}2 \times 10^4$
5×10^6	$5{\cdot}3 \times 10^5$
10^7	$1{\cdot}2 \times 10^6$

Only a limited number of dye studies have been made in which the horizontal spread of substance is compared with environmental factors such as the stability of a water column. FOXWORTHY, TIBBY and BARSOM (1966) examine the effect of water column stability and wind speed on the variances in the direction of the principal axes of a diffusing patch. At any given average value of wind speed ranging from 2 to 14 kt, lower values of dilution of dye are associated with higher values of average stability. This effect is controlled primarily by the suppression of vertical mixing by increasing stability. The effect of stability on horizontal diffusion is not so obvious, however. Thus, in the range of 2 to 8 kt wind, they suggest an increase in $\sigma^2{}_X$ with decreasing stability, but there are insufficient data on which to base a firm conclusion. Similar stability appears to have little or no definite effect on $\sigma^2{}_Y$. Although the data scatter considerably on a plot of the variances and wind speed, there is indicated a trend of increasing $\sigma^2{}_X$ with increasing wind speed, whereas there appears to be no definite correlation between $\sigma^2{}_Y$ and wind speed. A future study should include these environmental factors as parameters in the description of the diffusion diagram.

DISCUSSION

Any interpretation of the diffusion diagrams must remain provisional until the mechanisms or physical processes giving rise to diffusion are well understood. In Figs. 3 and 4, we have shown an interpretation based on the similarity theory of turbulence. However, this is by no means decisive.

The turbulent diffusion of a patch of dye is a problem in relative diffusion (BATCHELOR, 1950). With the aid of dimensional arguments, one finds a regime of relative diffusion where the inital size of the patch is infinitesimal. This regime is characterized by the following:

$$\sigma^2{}_{rc} = c_1 \epsilon t^3 , \tag{5}$$

where c_1 is a numerical constant. From (5) one can derive the relation between the apparent diffusivity and the scale of diffusion (the 4/3 power law):

$$K_a = c_2 \epsilon^{1/3} l^{4/3} \tag{6}$$

where c_2 is a numerical constant.

There are yet other possibilities considered (for example, see OZMIDOV, 1968; OKUBO and OZMIDOV, 1970). Even the dependence of our interpretation on the similarity theory of turbulence might be in error if some other processes operate on the diffusing patch. It has become apparent that shear in the mean flow can play an important role in the horizontal spread of substance in the sea (OKUBO, 1968). The basic result of current shear is that effective longitudinal dispersion is produced by the combination of the gradient of mean velocity and turbulent mixing in the same direction (BOWDEN, 1965). Thus, in the case of a uniform vertical shear in the horizontal current, the longitudinal variance is proportional to $\Omega K_z t^3$ where Ω denotes the vertical shear and K_z the vertical diffusivity. The implication is that the similarity theory of turbulence is not the *only* theory which deduces the third power law of the variance.

Recently some efforts have been made for understanding the mechanisms and processes which give rise to mixing in the upper boundary layer of the sea and lakes (FALLER, 1969; SCOTT, MYER, STEWART and WALTHER, 1969; ASSAF, GERARD and GORDON, 1971). All these recognize the importance of Langmuir circulations (LANGMUIR, 1938) in the mixing. Along this line of thought, however, we still have a long distance to the goal: the interpretation of the diffusion diagrams.

FINAL REMARKS

(1) The diffusion diagrams herein presented are made exclusively from the data of diffusion experiments in the surface layer of the sea. Very little has been known about the horizontal diffusion in deep water or in the thermocline simply because of the technical difficulties in carrying out dye release experiments there. Doubtless those experiments would provide important clues to our understanding of large-scale mixing in the sea.

(2) In order to provide adequate information on the study of oceanic diffusion, a particular manner of data reporting must be used. A good example of data presentation is seen in papers by JOSEPH, SENDNER and WEIDEMANN (1964) and by CARTER and OKUBO (1965). The following information should be considered as ' necessary ', whereas some of them have often been missing in reports:

(i) vertical distribution of concentration as a measure of depth of mixing,

(ii) velocity field, especially current shears in the scale of diffusion; a minimum requirement is the mean velocity of the centre of mass of patch during the interval of successive observations,

(iii) initial conditions of release: the duration of release, the depth of release, the initial patch size both horizontal and vertical, etc.,

(iv) shape information: the degree of elongation, the direction of elongation with respect to the mean flow, etc.; the pattern of the horizontal distribution of concentration should be shown whenever available,

(v) background oceanographic conditions; the stability of water, etc.,

(vi) wind data, sea state, etc.

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