The drag coefficient, bottom roughness, and wave-breaking in the nearshore

Falk Feddersen\textsuperscript{1}, E. L. Gallagher\textsuperscript{2}, R. T. Guza\textsuperscript{3}, and Steve Elgar\textsuperscript{1}

Short title: THE DRAG COEFFICIENT IN THE NEARSHORE

\textsuperscript{1}Woods Hole Oceanographic Institution, Woods Hole, Massachusetts.

\textsuperscript{2}Naval Postgraduate School, Monterrey, California

\textsuperscript{3}Center for Coastal Studies, Scripps Institution of Oceanography, La Jolla, California
**Abstract.** The bottom drag coefficient in the nearshore has been suggested to depend on bottom roughness (bedforms) or alternatively on wave breaking. The hypothesis that bottom drag coefficient depends on bottom roughness is tested with two months of field observations collected on a sandy ocean beach during the Duck94 field experiment. Both the drag coefficient (estimated from alongshore momentum balances) and bottom roughness (estimated from fixed altimeters) are larger within the surfzone than in the region farther seaward. Although the drag coefficient increases with roughness seaward of the surfzone, no relationship was found between the drag coefficient and roughness related quantities within the surfzone. These results suggest that breaking-wave generated turbulence increases the surfzone drag coefficient.

**Keywords:** Nearshore, Bed Roughness, Drag Coefficient, Circulation
1. Introduction

The mean (time-averaged) bottom stress is an important component of nearshore circulation and sediment transport dynamics. In depth-integrated circulation models, the mean alongshore bottom stress $\tau_b^y$ often is written as

$$\tau_b^y = \rho c_d \langle |\vec{u}|v \rangle$$

where $\rho$ is the water density, $c_d$ is the non-dimensional drag coefficient, and $\langle \cdot \rangle$ represents a time average over many wave periods. The horizontal velocity vector $\vec{u}$ and the alongshore velocity $v$ include both mean and wave components, above the bottom boundary layer. In nearshore circulation models, $\langle |\vec{u}|v \rangle$ can be represented well with low-order moments of the velocity field [Feddersen et al., 2000], and thus accurate parameterizations of $c_d$ are required to model the bottom stress.

The bottom stress is equal to the turbulent vertical flux of horizontal momentum into a viscous bottom boundary layer, i.e., for the alongshore bottom stress,

$$\tau_b^y = -\rho \langle v'w' \rangle$$

where $v'$ and $w'$ are the turbulent alongshore and vertical velocities, respectively. Therefore, $c_d$ depends on the turbulence, and for constant $\langle |\vec{u}|v \rangle$, $c_d$ increases with increased turbulence levels. Both of the two primary sources of nearshore turbulence, shear in the bottom boundary layer [e.g., Grant and Madsen, 1979], and the breaking of surface gravity waves [e.g., Svendsen, 1987], have been proposed to affect $c_d$ [e.g., Fredsoe and Deigaard, 1992; Van Rijn, 1993]. For simplicity, many nearshore circulation models have assumed a spatially
constant $c_d$ [Longuet-Higgins, 1970; Özkan-Haller and Kirby, 1999, and many others], with the value of $c_d$ usually determined by fitting to observations.

On the continental shelf, breaking wave generated turbulence does not reach the lower part of the water column and thus does not influence the bottom boundary layer. Grant and Madsen [1979] generalized the Prandtl-Karman law of the wall to the continental shelf bottom boundary layer in the presence of wave-orbital velocities and bottom roughness, i.e.,

$$
\overline{v}(z) = \frac{\overline{v}_*}{\kappa} \log \left( \frac{z}{z_a} \right),
$$

where $z$ is the height above the bottom, $z_a$ is the apparent roughness height that depends on waves and bottom roughness, and $\kappa$ is von Karman constant. The current friction velocity $\overline{v}_*$ is defined so that

$$
\tau^v_b = \rho \overline{v}_*^2.
$$

Garcez-Faria et al. [1998] used this model to estimate the alongshore bottom stress in the nearshore (depths $< 4$ m) by fitting alongshore current observations that spanned much of the water column to a log profile (3), and solved for $c_d$ using (4) and (1). Garcez-Faria et al. [1998] found that $c_d$ was related to the root-mean-square bottom roughness normalized by water depth $k_{rms}/h$ with correlation $r = 0.63$, and that $c_d$ was inversely proportional to the percentage of waves breaking. In contrast, Fredsoe and Deigaard [1992] and Church and Thornton [1993] hypothesized that differences in $c_d$ within and seaward of the surfzone are caused by differences in breaking-wave turbulence levels, with increased breaking resulting in larger $c_d$. Feddersen et al. [1998] found larger (by factor of 3) $c_d$ within the surfzone relative to seaward of the surfzone, but it was unclear whether this $c_d$ variation resulted from differences
in bottom roughness or wave breaking.

Here, the dependence of nearshore $c_d$ on bottom roughness and wave breaking is examined further using two months of observations acquired on a sandy ocean beach. Bottom roughness observations [Gallagher et al., 1998a] obtained concurrently with wave and current observations [Feddersen et al., 1998] are used to estimate spatial averages of roughness quantities ($k_{rms}$ and $k_{rms}/h$) and $c_d$, as described in section 2. Although $c_d$ and roughness variables are consistently larger within the surfzone than seaward of the surfzone, no relationship is observed between $c_d$ and roughness variables within the surfzone (section 3). This suggests that in these observations the elevated surfzone $c_d$ likely is influenced more strongly by breaking-wave generated turbulence than by elevated bottom roughness, consistent with the hypothesis of Fredsoe and Deigaard [1992] and Church and Thornton [1993]. Reasons these results differ from those of Garcez-Faria et al. [1998], in particular the limitations of the methods used here and of the log-profile approach (3) in the surfzone, are discussed in section 4.

2. Observations and Methods

Observations were obtained during the Duck94 field experiment (September–October, 1994) near Duck, North Carolina. Pressure sensors, bidirectional current meters, and altimeters, sampled at 2 Hz, were deployed on a cross-shore transect (Figure 1) extending 750 m from near the shoreline to 8-m water depth [Gallagher et al., 1998a; Feddersen et al., 1998].

Figure 1.
Roughness Estimates

The altimeters measure acoustically the distance to the seafloor from a fixed frame. At
each altimeter, the 2 Hz data were processed into 32-s bed-location estimates (e.g., Figure 2)
[Gallagher et al., 1996], that were demeaned and detrended to produce 24-hr root-mean-square
(rms) bed roughness $k_{\text{rms}}$ estimates. Bed roughness normalized by water depth $k_{\text{rms}}/h$ also
was estimated every 24 hours, using the 24-hr average water depth $h$.

This $k_{\text{rms}}$ estimation method assumes that the bedforms migrate under the altimeter so
that time variability approximates spatial variability. If either the roughness field is frozen or
if the bed erodes or accretes uniformly in space (at time scales not removed by detrending),
this method fails. The $k_{\text{rms}}$ estimates are believed accurate for the following reasons. First,
coherently migrating bedforms were observed 60% of the time under a 1.4 m by 1.4 m
altimeter array co-deployed 70 m from the shoreline [Gallagher et al., 1998b], supporting the
assumption that time variability approximates spatial variability. For most of the remaining
40% of the time, $k_{\text{rms}}$ was small. Second, the magnitude and variability of $k_{\text{rms}}$ in the
cross-shore (Figure 3) is consistent with spatial-series based $k_{\text{rms}}$ observations occasionally
collected during this experiment [Thornton et al., 1998]. The mean and variability of $k_{\text{rms}}$ are
largest within 100 m of the shoreline (where the surfzone usually is located) and decay farther
offshore. Third, towed and fixed altimeter $k_{\text{rms}}$ estimates agree well within and seaward of the
surfzone at the same beach during an experiment three years later (not shown).

Spatially weighted (i.e., integral) averages of $k_{\text{rms}}$ and $k_{\text{rms}}/h$ were calculated on the
transect within and seaward of the surfzone (Figure 1). The cross-shore extent of the surfzone
(averaged over 24 hours) was estimated heuristically based on energy flux relative to the flux in 8-m depth, the local energy flux gradient, and time-lapsed video images (R. A. Holman, personal communication, 1996) as described in Feddersen et al. [1998]. The 24-hr \( k_{\text{rms}} \) are significantly (> 98%) correlated at cross-shore lags up to 125 m, indicating that the cross-shore \( k_{\text{rms}} \) variability is not dominated by unresolved short spatial scales that would cause errors in the spatial averages. The 24-hr averaged roughness quantities (\( k_{\text{rms}} \) and \( k_{\text{rms}}/h \)) for the full-transect, the surfzone, and seaward of the surfzone were averaged into 48-hr estimates for comparison with \( c_d \) estimates.

Bedform lengthscales and orientation cannot be determined from these altimeter observations. During the same experiment, the dominant horizontal spatial scales of bed variability were in the range 1–5 m [Thornton et al., 1998; Gallagher et al., 1998b], similar to the observed wave-orbital diameters (1–4 m). The altimeter-estimated \( k_{\text{rms}} \) are assumed to correspond to the bedforms with these lengthscales. The orientation of long-crested bedforms relative to a steady current can have a significant effect on the bottom stress [Barrantes and Madsen, 2000]. However, the effect of bedform orientation and lengthscales in the surfzone with combined wave–current flows is not understood. Thus, as in Garcez-Faria et al. [1998], the relationship between bedform height and \( c_d \) is investigated, and the effects of bedform lengthscales and orientation while potentially significant are not considered.
Drag Coefficient Estimates

Drag coefficient estimates are based on a 1-D alongshore momentum balance between wind and wave forcing, bottom stress, and lateral mixing given by

$$\tau_y^{\text{wind}} - \frac{dS_{xy}}{dx} = \rho c_d \langle u'v' \rangle + \frac{dM_{xy}}{dx},$$  

(5)

where $\tau_y^{\text{wind}}$ is the alongshore ($y$) wind stress. Cross-shore ($x$) derivatives of $S_{xy}$ and $M_{xy}$, components of the radiation and depth-integrated lateral Reynolds stresses [e.g., Svendsen and Putrevu, 1994], respectively, are difficult to estimate with observations. However, the observations can be used to estimate cross-shore integrals of terms in (5), from which spatially-averaged surfzone and seaward of the surfzone $c_d$ can be calculated.

The $c_d$ within and seaward of the surfzone are calculated by integrating (5) over the entire 750-m long current meter transect from approximately the shoreline ($x = 0$) to 8-m water depth ($x = x_{8m}$) [Feddersen et al., 1998]. The cross-shore integration is separated into two components, one spanning the surfzone and one seaward of the surfzone. In 8-m water depth, usually well seaward of the surfzone, pressure array data and linear theory are used to estimate $S_{xy}$, and the Reynolds stress $M_{xy}$ is assumed negligible. At $x = 0$, the location of the most shoreward instrument, $S_{xy}$ and $M_{xy}$ are set equal to zero. Swash processes onshore of the most shoreward current meter are thus assumed to contribute negligibly to the cross-shore integrated, alongshore momentum balance. This assumption is consistent with standard models for radiation (based on depth-limited wave breaking) and Reynolds stresses, and is supported by the closure of an integrated alongshore momentum balance that neglects the swash region [Feddersen et al., 1998]. Although $c_d$ may vary continuously in the cross-shore,
$c_d$ is assumed spatially constant within each the surfzone and seaward of the surfzone regions, and $c_d$ is passed through the integrals. With a spatially constant wind stress, the cross-shore integral of (5) becomes,

$$
\tau_{y, \text{wind}} \cdot x_{\text{sm}} - S_{xy, \text{sm}} = \rho c_{d1} \int_0^{x_b} \langle |u| v \rangle \, dx + \rho c_{d2} \int_{x_b}^{x_{\text{sm}}} \langle |u| v \rangle \, dx,
$$

(6)

where $x_b$ is the location of the breakpoint. The unknown drag coefficients within and seaward of the surfzone are represented by $c_{d1}$ and $c_{d2}$, respectively. Observed 2-Hz velocity time series are used to estimate $\langle |u| v \rangle$ at each current meter. Hourly estimates of the total forcing, and surfzone and seaward integrated $\langle |u| v \rangle$ are linearly regressed to calculate 48-hr values of best-fit $c_d$ within and seaward of the surfzone (see Feddersen et al. [1998] for details).

The selected 48-hr time interval is a compromise between long time intervals needed for statistical stability of the $c_d$ estimates, and short-time intervals that better resolve temporal variability of $k_{\text{rms}}, c_d$, and the cross-shore extent of the surfzone. The results were similar using 24- and 48-hr averaging intervals. Successive hourly estimates of integrated $\langle |u| v \rangle$ (and of the total forcing) are not independent [Feddersen et al., 1998], and thus the effective degrees of freedom and confidence limits for each 48-hr $c_d$ estimate cannot be determined. To eliminate inaccurate estimates, 48-hr $c_d$ values were rejected if the regression had poor skill (defined as skill $< 0.4$) or if $c_d$ was negative.
3. Relationship Between the Drag Coefficient, Roughness, and Wave Breaking

Estimates of \( c_d \), \( k_{\text{rms}} \), and \( k_{\text{rms}}/h \) are consistently larger in the surfzone than seaward of the surfzone (compare circles with crosses in Figure 4). The average surfzone \( c_d \) is significantly (> 99% confidence) larger than the average \( c_d \) seaward of the surfzone, consistent with \( c_d \) estimated using a single regression for the entire 2-month period (dashed lines in Figure 4), but roughly a factor of two less than the equivalent \( c_d \) derived from friction factors cited by Nielsen et al. [2001].

A relationship between \( k_{\text{rms}} \) and \( c_d \) (Figure 4a) and \( k_{\text{rms}}/h \) and \( c_d \) (Figure 4b) is apparent when the regions within and seaward of the surfzone are considered together. However, this is misleading because considering both regions together does not control for other factors that could effect \( c_d \) such as breaking-wave generated turbulence.

To isolate the effect of enhanced turbulence due to bottom roughness from turbulence due to breaking waves, the relationship between roughness quantities and \( c_d \) is examined separately within and seaward of the surfzone. Within the surfzone, no relationship is observed between \( k_{\text{rms}} \) and \( c_d \) (circles in Figure 4a), nor between \( k_{\text{rms}}/h \) and \( c_d \) (circles in Figure 4b). The correlations (Table 1) are not significant at the 90% confidence level. The lack of a detectable \( c_d \) dependence on spatially averaged \( k_{\text{rms}} \) or \( k_{\text{rms}}/h \) suggests that roughness quantities are not the critical factor determining the surfzone \( c_d \).

Seaward of the surfzone the correlation between \( k_{\text{rms}} \) and \( c_d \) (\( r = 0.33 \), Table 1, crosses in Figure 4a) is increased relative to the correlation within the surfzone, but is not significant.
at the 90% level, suggesting that $k_{\text{rms}}$ alone is not responsible for the $c_d$ variation seaward of the surfzone. However, the correlation ($r = 0.47$, Table 1) between seaward of the surfzone $k_{\text{rms}}/h$ and $c_d$ is significant at the 90% level (crosses in Figure 4b). This is consistent with the hypothesis that $c_d$ seaward of the surfzone depends on the depth-normalized apparent roughness $k_a/h$, because $k_a$ is a function of the physical roughness [Grant and Madsen, 1979].

4. Discussion

The consistently elevated surfzone $c_d$ over a broad range of roughness (Figure 4) implies that other surfzone processes, such as wave breaking, are important to $c_d$. However, this conclusion is tentative due to limitations of the data and analysis methods. These limitations include the inability to estimate bed roughness more frequently than every 24 hours because bedforms migrate slowly past the altimeter (Figure 2). Using 24-hr averaged roughness quantities and breakpoint locations can degrade roughness estimates within and seaward of the surfzone by including observations from locations sometimes outside each region when there is a rapid change in surfzone width. Also, the instantaneous bed roughness is spatially patchy [Gallagher et al., 2002], which is obscured with a 24-hr averaged $k_{\text{rms}}$. The potential effect of variable bedform lengths and orientations [Barrantes and Madsen, 2000] were not taken into account. In addition, the estimated correlation between $c_d$ and roughness will be reduced below the true correlation if $c_d$ and roughness vary over the 48-hr averaging time necessary to obtain statistical stability.

The result that $c_d$ does not depend on roughness in the surfzone differs from the (log profile based) result of Garcez-Faria et al. [1998]. A requirement of log-profile models (3)
is that the bottom boundary layer is a constant stress layer where shear production balances turbulent dissipation $\epsilon$, yielding a dissipation scaling

$$\epsilon = \frac{\bar{v}^3}{\kappa z}$$  \hspace{1cm} (7)$$

where $\epsilon$ decreases with height $z$ above the bed. This dissipation scaling (7) is consistent with measurements (for example) on the Northern California continental shelf [Grant et al., 1984] and the Hudson River Estuary [Trowbridge et al., 1999].

In the shallow water of the surfzone, breaking-wave generated turbulence can penetrate the entire water column. For example, laboratory observations show increased turbulence associated with wave breaking within 0.3 cm of the bed [Cox and Kobayashi, 2000]. Surfzone dissipation observed in the field [George et al., 1994] is $10^2$ to $10^3$ times larger than the near-bottom dissipation observed in a tidal estuary [Trowbridge et al., 1999], and does not decay as $z^{-1}$ (7), but increases with height above the bed. Laboratory measurements of surfzone turbulent kinetic energy $q$ show a maximum at mid-water column [Ting and Kirby, 1994]. If $\epsilon \sim q^{3/2}$, as commonly is assumed in the surfzone [e.g., Svendsen, 1987], the laboratory surfzone $\epsilon$ also increases with height above the bed.

These (and other) laboratory and natural surfzone observations suggest that the dissipation scaling (7) likely is not applicable in the surfzone. Thus, mean bottom stress (and therefore $c_d$) inferred by fitting mean currents to a log profile may be incorrect. Similarly, the concept of an apparent roughness $k_a$ [e.g., Grant and Madsen, 1979] also is not applicable to the surfzone, and the relationship between $k_a$ and $c_d$ therefore is not investigated.
5. Conclusions

Observations along a cross-shore transect of current meters and altimeters extending 750 m from the shoreline to 8-m water depth show that the drag coefficient $c_d$ and bottom roughness $k_{rms}$ are larger inside the surfzone than outside the surfzone. No dependence of $c_d$ on $k_{rms}$ or $k_{rms}/h$ is found within the surfzone, nor between $c_d$ and $k_{rms}$ seaward of the surfzone. There is a weak relationship between $c_d$ and $k_{rms}/h$ seaward of the surfzone, consistent with the hypothesis that $k_{rms}/h$ influences $c_d$ when waves are not breaking. Although the data and methods have limitations, the lack of an observed relationship between $c_d$ and roughness, together with the existing evidence of increased surfzone turbulence dissipation associated with breaking waves, suggest that breaking-wave generated turbulence leads to increased surfzone $c_d$.

Acknowledgments. The instrument array was deployed and maintained by staff from the Center for Coastal Studies. Tom Herbers and Britt Raubenheimer helped collect and process the data. Excellent logistical support and the 8-m depth pressure sensor array data were provided by the U.S. Army Corps of Engineers Field Research Facility. Steve Lentz provided the wind stress data. We have benefited from discussions with John Trowbridge, Tony Bowen, Rob Holman, and an anonymous reviewer. This research was funded by the Office of Naval Research, the National Science Foundation, the National Ocean Partnership Program, and the Army Research Office. Woods Hole Oceanographic Institution contribution 10286.
References


Thornton, E. B., J. L. Swayne, and J. R. Dingler, Small-scale morphology across the surf zone, 


F. Feddersen and S. Elgar, Woods Hole Oceanographic Institution, AOPE MS#12, Woods Hole, MA, 02543. (e-mail: falk@whoi.edu; elgar@whoi.edu)

R. T. Guza, Center for Coastal Studies Scripps Institution of Oceanography, University of California, La Jolla, CA 92093-0209. (e-mail: rguza@ucsd.edu)

E. L. Gallagher, Dept. of Oceanography, Naval Postgraduate School, Monterey, CA 93943-5122 (e-mail: egallagh@oc.nps.navy.mil)

Received March 17, 2003

This manuscript was prepared with AGU’s LATEX macros v5, with the extension package ‘AGU++’ by P. W. Daly, version 1.6b from 1999/08/19.
**Figure Captions**

![Figure 1](image1.png)

**Figure 1.** Depth versus distance from the shoreline (solid curve) on August 25, 1994, and locations of current meters (circles) and altimeters (crosses). An additional current meter and the pressure sensor array were located 750 m from the shoreline in approximately 8-m water depth.

![Figure 2](image2.png)

**Figure 2.** Demeaned 32-s bed elevation versus time for a 48-hr period. The $k_{\text{rms}}$ is 6.9 and 4.5 cm for the first and second 24-hr periods, respectively.
Figure 3. Mean (diamonds) and standard deviation (vertical bars) of $k_{\text{rms}}$ versus distance from the shoreline. There are between 53 and 58 24-hr observations at each location. The mean for all 511 24-hr observations is 2.9 cm, the standard deviation is 2.2 cm, and the largest observed $k_{\text{rms}}$ is 10.7 cm.
Figure 4. (a) $c_d$ versus $k_{\text{rms}}$ and (b) $c_d$ versus $k_{\text{rms}}/h$ for the surfzone (circles) and seaward of the surfzone (crosses). The upper and lower dashed lines in each panel are the surfzone and seaward of the surfzone two-month best-fit $c_d$, respectively [Feddersen et al., 1998]. Statistics are presented in Table 1.
**Tables**

**Table 1.** Correlation $r$ between drag coefficient $c_d$ and both bottom roughness $k_{rms}$ and normalized bottom roughness $k_{rms}/h$. The number of data points in each correlation is $N$.

<table>
<thead>
<tr>
<th>Seaward of the Surfzone</th>
<th>$N$</th>
<th>$r$</th>
<th>$N$</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{rms}$</td>
<td>17</td>
<td>0.27</td>
<td>15</td>
<td>0.33</td>
</tr>
<tr>
<td>$k_{rms}/h$</td>
<td>13</td>
<td>0.23</td>
<td>11</td>
<td>0.47†</td>
</tr>
</tbody>
</table>

†Correlation significant ($\neq 0$) with 90% confidence