A self-similar scaling for cross-shelf exchange driven by transient rip currents

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Transient rip currents, episodic offshore flows from the surfzone to the inner-shelf, present a recreational beach hazard and exchange material across the nearshore ocean. The magnitude and offshore extent of transient rip current-induced exchange and its relative importance to other inner-shelf exchange processes is poorly understood. Here, 120 model simulations with random, normally-incident directionally spread waves spanning a range of beach slopes and wave conditions show that the transient rip current exchange velocity is self-similar. The nondimensional exchange velocity, surfzone flushing time, and cross-shore decay length-scale are scaled by beach slope and wave properties, depending strongly on wave directional spread. Transient rip current driven exchange can be compared to other cross-shelf exchange processes. For example, transient rip current driven exchange is stronger than wave-induced Stokes’ drift driven exchange up to six surfzone widths from shore.
1. Introduction

The nearshore, the $\approx 1$ km of ocean adjacent to the coastline, consists of the surfzone, where circulation is forced by breaking surface waves, and the inner shelf immediately offshore, where flows are driven by winds, buoyancy, tides, and non-breaking waves [Dalrymple et al., 2011; Lentz and Fewings, 2012]. Understanding cross-shelf exchange between the surfzone and inner shelf is challenging due to these dynamical differences yet essential to many areas of societal concern such as maintaining beaches, pollution dispersal, and managing coastal ecosystems. For example, tracers (e.g., terrestrial pollutants), introduced at the shoreline, are diluted in the surfzone by exchange with the inner shelf [e.g., Hally-Rosendahl et al., 2014]. Additionally, the recruitment of intertidal invertebrate larvae depends on cross-shelf exchange processes [e.g., Shanks and Wright, 1987].

A variety of physical processes can induce cross-shelf exchange. On the inner shelf, the vertical mismatch between offshore directed undertow and onshore wave-driven Stokes drift results in cross-shelf exchange [Lentz et al., 2008; Kirincich et al., 2009; McPhee-Shaw et al., 2011]. At subtidal time-scales, cross-shelf winds can induce cross inner-shelf exchange particularly relative to along-shelf winds [Fewings et al., 2008]. Shoreward-propagating nonlinear internal waves [e.g., Pineda, 1991; Sinnett and Feddersen, 2014] and shelf eddies [e.g., Romero et al., 2013; Uchiyama et al., 2014] can also exchange material across the inner-shelf.

Rip currents, concentrated, offshore-directed flows from the surfzone to the inner shelf, exchange sediments [Shepard et al., 1941], pollutants [Boehm et al., 2005], and larvae [Shanks et al., 2010]. Rip currents are also a beach hazard accounting for 80% of U.S. life guard rescues, with more than 100 swimmer drownings each year [MacMahan et al., 2006, 2010]. Rip currents
are classified as bathymetrically-controlled or transient. Bathymetrically-controlled rip currents occur at fixed alongshore locations near structures or on embayed or rip-channeled beaches, are relatively steady, and can be considered a component of the mean circulation [MacMahan et al., 2010; Dalrymple et al., 2011]. Modeling [Reniers et al., 2010; Castelle and Coco, 2013] and observations [Brown et al., 2015] indicate that bathymetrically controlled rips can exchange material 2–4 surfzone widths from the shoreline.

In contrast, transient rip currents occur on alongshore uniform bathymetry and originate from surfzone eddies generated by finite-crest length wave breaking which then coalesce to larger scales [Peregrine, 1998; Johnson and Pattiaratchi, 2006; Spydell and Feddersen, 2009; Clark et al., 2012]. Transient rip currents occur episodically for $O(1 \text{ min})$, have short (10 - 100 m) alongshore length scales [Hally-Rosendahl et al., 2014], and no preferred alongshore location [Dalrymple et al., 2011] (Figure 1a). Thus, transient rip currents can be considered two-dimensional turbulence as their length scales are much longer than the water depth [Feddersen, 2014]. Although transient rip currents are ubiquitous on alongshore uniform beaches, few observational studies have attempted to quantify transient rip current exchange due to their episodic nature [Johnson and Pattiaratchi, 2004; Hally-Rosendahl et al., 2014].

Previously, surfzone eddy intensity was shown to increase with wave directional spread $\sigma_\theta$ for the same significant wave height $H_s$ and peak period $T_p$ [Spydell and Feddersen, 2009]. As transient rip currents result from surfzone eddies, these wave parameters should also determine the exchange velocity from the surfzone to inner shelf representative of rip currents. Here, cross-shelf exchange by transient rip currents is investigated with 120 simulations using a wave-resolving model *funwaveC*, spanning a range of normally-incident wave conditions and beach
slopes. The model setup and methods for estimating transient rip current exchange are presented (Section 2). The cross-shore profile of transient rip current exchange is self-similar and is scaled by the incident wave conditions, particularly wave directional spread (Section 3). The implications of transient rip currents on surfzone residence time and the relative importance of transient rip current driven exchange to wave-driven Stokes drift exchange on the inner-shelf are discussed in Section 4. The results are summarized in Section 5.

2. Model and Methods

The wave-resolving model funwaveC [Spydell and Feddersen, 2009; Feddersen et al., 2011; Feddersen, 2014] solves the Boussinesq mass and momentum equations with nonlinear and dispersive effects [Nwogu, 1993], and parameterized wave breaking [Kennedy et al., 2000]. In all simulations, the bathymetry is alongshore uniform (domain length, $L_y = 1200$ m), with an offshore flat region (depth, $h = 9$ m) where waves are generated and a planar slope region extending above the mean water line allowing wave runup. A source function method [Wei et al., 1999] generates normally-incident random waves from a Pierson-Moskovitz spectrum [Pierson and Moskowitz, 1964], characterized by significant wave height $H_s$ and peak period $T_p$. The normally incident waves have directional spread $\sigma_\theta$ [Kuik et al., 1988] with a Gaussian shape that is uniform at all frequencies.

A total of 120 model simulations are performed spanning a range of beach slopes ($\beta = 0.02, 0.03, 0.04, 0.05, 0.06$) and wave parameters significant wave height ($H_s = 0.5, 0.8, 1.1$ m), peak period ($T_p = 8, 14$ s), and wave directional spread ($\sigma_\theta = 2.5^\circ, 5^\circ, 10^\circ, 20^\circ$). Simulations were run for 8000 s, with the last 6000 s used for analysis once mean square vorticity has
equilibrated [Feddersen, 2014]. Standard analyses [Kuik et al., 1988] estimate $H_s(x)$ and bulk $\sigma_\theta(x)$.

Model horizontal velocities are decomposed into rotational (eddies, rip currents) and irrotational (wave) components [e.g., Spydell and Feddersen, 2009]. The net cross-shelf exchange due to transient rip currents is quantified using a rip current exchange velocity ($U_{ex}^r$), representative of the time- and alongshore-averaged exchange from the offshore-directed (negative) component of the rotational flow $u_{rot}^-$, i.e.,

$$U_{ex}^r(x) = \left\langle \frac{1}{L_y} \int u_{rot}^-(x, y, t) dy \right\rangle,$$

(1)

where $\langle \rangle$ is a time-average. The definition of $U_{ex}^r$ (1) is analogous to the definition of estuarine total exchange flow [MacCready, 2011]. In calculating (1), the time-mean of $u_{rot}^-$ is removed to eliminate any potential standing rip current structures [i.e., Johnson and Pattiaratchi, 2006]. However, retaining the mean does not affect the results.

3. Results

An example simulation is shown in Figure 1. As random, normally incident and directionally-spread waves shoal propagate over the bathymetry ($h$, Figure 1e), $H_s$ increases to a maximum, defining breakpoint location (Figure 1c) delimiting the surfzone (width $L_{sz} = 97$ m) and inner shelf. Quantities located at the breakpoint are denoted by subscript “$b$” (e.g., $H_{sb}$). Onshore of the breakpoint, $H_s$ decays towards the shoreline as finite-crest length wave breaking generates vertical vorticity (eddies) [Peregrine, 1998; Johnson and Pattiaratchi, 2006; Clark et al., 2012]. These eddies coalesce to larger scales [Spydell and Feddersen, 2009; Feddersen, 2014], resulting in episodic transient rip currents [Johnson and Pattiaratchi, 2006] with a vortex dipole signature and strong velocities, here $> 1$ m s$^{-1}$ at $x = L_{sz}$ (Figure 1b). In this example, the
maximum ($U_{r ex0} = 0.06 \text{ m s}^{-1}$) occurs at $x \approx L_{sz}/2$ (Figure 1d), decays as a Gaussian offshore, and remains significant ($> 1 \text{ cm s}^{-1}$) up to 100 m beyond $L_{sz}$. As $U_{r ex}$ represents net exchange, it is far weaker than the $O(1) \text{ m s}^{-1}$ maximum flow in an individual rip current (Figure 1b).

Similar to the example (Figure 1), over a range of $\beta$ and wave conditions, $U_{r ex}(x)$ profiles are approximately Gaussian with a maximum between 0.01–0.1 m s$^{-1}$ and a cross-shore decay scale $L_d$ of 50–250 m, comparable to the surfzone width $L_{sz}$ (Figure 2a). The $U_{r ex}$ cross-shore profiles are well-fit (regression skill $> 0.8$) to a Gaussian form,

$$U_{r ex}(x) = U_{r ex0} \exp \left[ -\frac{(x - x_o)^2}{L_d^2} \right]$$

yielding three $U_{r ex}$ fit parameters ($U_{r ex0}$, $x_o$, $L_d$) which are best-fit using iterative least-squares for each simulation.

The nondimensionalized $U_{r ex}/U_{r ex0}$ versus $(x - x_o)/L_d$ profiles collapse into a self-similar form (Figure 2b), suggesting a scaling law for transient rip current driven exchange velocity $U_{r ex}(x)$. Here, the three exchange parameters ($U_{r ex0}$, $x_o$, $L_d$) are nondimensionalized and scaled by the incident wave parameters and beach slope $\beta$ with the following nondimensional scalings (Figure 2c–e)

$$\frac{U_{r ex0}}{(gh_{\beta})^{1/2}} = \sigma_{\theta_b} \left[ 0.035S_{\beta} + 5 \times 10^{-4} \right],$$

$$\frac{x_o}{L_{sz}} = \left[ 2 \times 10^{-3}S_{\beta}^{-1} + 0.39 \right],$$

$$\frac{L_d}{L_{sz}} = \text{Ir}_{\infty} \sigma_{\theta_b}^{-0.25} \left[ 35.24S_{\infty}^{0.5} + 1.59 \right].$$

Nondimensional surfzone variables are the wave steepness ($S = H_s/L$, with $L$ the local wavelength given from $T_p$ and the wave phase speed) evaluated in deep water ($S_{\infty}$) or at the break-point ($S_{\beta}$), and deep-water Irribarren number [Battjes, 1974] $\text{Ir}_{\infty} = \beta/S_{\infty}^{1/2}$. The maximum exchange velocity $U_{r ex0}$ is scaled by the breakpoint shallow water wave phase speed $(gh_{\beta})^{1/2}$. 
The nondimensional $U_{ex0}^r (gh_b)^{-1/2}$ depends strongly on the directional spread at the breakpoint $\sigma_{b\theta}$ with residual $S_b$ dependence (Figure 2c). The strong $U_{ex}^r$ dependence on wave directional spread $\sigma_{b\theta}$ highlights the important role of finite-crest length wave breaking [Peregrine, 1998; Clark et al., 2012] in generating surfzone vorticity that develops into transient rip currents.

The nondimensional location of the maximum $x_o/L_{sz} \approx 0.5$ has weak $S_{b}^{-1}$ dependence (Figure 2d). The nondimensional cross-shore decay scale $L_d/L_{sz}$ largely varies between 0.9–2.5.

With $x_0/L_{sz} \approx 0.5$, the $U_{ex}^r$ e-folding decay location is 0.4–1.9 $L_{sz}$ offshore of the surfzone to inner-shelf boundary. The nondimensional $L_d/L_{sz}$ is a strong function of $I_{r\infty}\sigma_{b\theta}^{-1/4}$ with a subsequent weak $S_{\infty}$ dependence (Figure 2e). Similar scaling results apply for root-mean-square rotational velocity.

Using the scalings (Eq. 3a–3c), the three scaled $U_{ex}^r$ parameters (denoted with a tilde; $\tilde{U}_{ex0}^r$, $\tilde{x}_o$, $\tilde{L}_d$) can be estimated from the wave parameters and $\beta$ directly. Though more scattered than the nondimensional profiles (Figure 2b), the scaled profiles $U_{ex}^r/\tilde{U}_{ex0}^r$ versus $(x - \tilde{x}_o)/\tilde{L}_d$ also largely collapse with small standard deviation relative to the mean ($< 15\%$, not shown).

All wave parameters ($H_s$, $T_p$, $\sigma_\theta$) can be predicted with modern spectral wave models, and the scaling laws can be used to estimate cross-shelf transient rip current exchange.

4. Discussion

Analogous to estuarine flushing time [MacCready, 2011; Lemagie and Lerczak, 2014], a bulk surfzone flushing time is useful in understanding surfzone to inner-shelf material exchange. Here, the transient rip current surfzone flushing time $T_f^r$ is defined as the time required to replace the surfzone area ($A_{sz} = h_bl_{sz}/2$),

$$T_f^r = \frac{A_{sz}}{h_bl_{ex}^r(L_{sz})} = \frac{L_{sz}}{2 \tilde{U}_{ex}^r(L_{sz})}.$$  

(4)
where $U_{\text{ex}}^r$ is evaluated at the surfzone inner-shelf boundary ($x = L_{sz}$). Short flushing times represent more rapid exchange of material and are a potential beach hazard as more bathers will be swept offshore. For all simulations, the dimensional $T_f^r$ generally varies between 20 min and 3 hr. Based on the $U_{\text{ex}}^r$ scalings (Eq. 3a–3c), the nondimensional flushing time $T_f^r (g/h_b)^{1/2} \beta$ varies more than an order of magnitude and depends proportionally on $\sigma^{-1}$ (Figure 3). The dimensional surfzone flushing time $T_f^r \propto (h_b/g)^{1/2} (\beta \sigma_{gb})^{-1}$ is thus larger for deeper breakpoint depths $h_b$, shallower surfzone slopes, and smaller directional spreads. With $\sigma_{gb} = 0^\circ$, no transient rip currents are generated and $T_f^r$ is infinite.

Transient rip current driven exchange is also estimated by advecting randomly-seeded surfzone particles. The e-folding time for particles to leave the surfzone also yields a flushing time and, with (4), an exchange velocity. This velocity is well-correlated with $U_{\text{ex}}^r (L_{sz}) (r^2 = 0.76)$ with a regression slope of 5 due to particle recirculation typically accounted for by an exchange factor [Lemagie and Lerczak, 2014]. Thus, $U_{\text{ex}}^r (L_{sz})$ is representative of the surfzone flushing rate and the scalings (3) allow the surfzone flushing time to be quantified via (4).

On alongshore uniform coasts, wave-driven exchange across the inner shelf is usually attributed to the imbalance between vertical profiles of onshore wave-driven Stokes’ drift transport and offshore directed Eulerian return flow [Monismith and Fong, 2004; Lentz et al., 2008; Lentz and Fewings, 2012], defining a Stokes’ exchange velocity $U_{\text{st}}^r$. The depth-integrated model fun-waveC does not resolve vertical velocity structure. Instead, as inner-shelf Eulerian return flow (undertow) is observed to be largely depth-uniform [Putrevu and Svendsen, 1993; Faria et al., 2000], Stokes’ drift induced exchange velocity $U_{\text{st}}^r(x)$ are estimated assuming onshore Stokes’
Drift balanced by a depth-uniform offshore flow [Hally-Rosendahl et al., 2014],

\[ U_{ex}^{st} = \frac{1}{8} (H_s k_p)^2 \frac{c}{\sinh^2 (k_p h)} \left[ \frac{1}{h} \int_{z_c}^{0} \left( \frac{\cosh(2k_p(z+h))}{2k_p h} - \frac{\sinh(2k_p h)}{2k_p h} \right) dz \right], \quad (5) \]

where \( k_p \) is the peak wavenumber, \( c \) is the linear phase speed, and the integral is from the integrand zero crossing \( (z_c) \) to the surface. Note, this \( U_{ex}^{st} (5) \) may be an overestimate if offshore Eulerian return flow is surface intensified [Lentz et al., 2008].

Previously inner shelf drifter velocities near the surfzone have been attributed to Stokes’ drift [Ohlmann et al., 2012]. Here, transient rip current \( (U_{ex}^{r}) \) and Stokes’ drift \( (U_{ex}^{st}) \) driven exchange velocities are compared to determine the offshore extent of transient rip current importance relative to Stokes’ drift. For the simulation shown in Figure 1, at the breakpoint \( (L_{sz} = 97 \text{ m}) \), \( U_{ex}^{r} \approx 0.04 \text{ m s}^{-1} \), much larger than \( U_{ex}^{st} \approx 0.002 \text{ m s}^{-1} \). Farther offshore \( U_{ex}^{r} \) decays more rapidly than \( U_{ex}^{st} \) where they become equivalent at \( X_I = 353 \text{ m} \) with magnitude \( 8 \times 10^{-4} \text{ m s}^{-1} \) (Figure 4a). The nondimensional location (relative to the shoreline) where \( U_{ex}^{r} = U_{ex}^{st} \) (defined as \( X_I / L_{sz} \)) generally varies from 2–6 and is linearly scaled with the breaking Irribarren number \( \text{Ir}_b = \beta / S_b^{1/2} \) (Figure 4b). This indicates that transient rip current driven exchange is larger than the Stokes’ exchange up to 2–6 surfzone widths from shore well onto the inner-shelf, in regions previously not thought to be influenced by the surfzone.

5. Summary

The wave-resolving model funwaveC, was used to simulate the cross-shore exchange induced by transient rip currents for a variety of normally-incident, directionally-spread random waves and beach slopes. The cross-shore profile of transient rip current exchange velocity is self-similar whose maximum magnitude, peak location, and cross-shore decay lengthscale is accurately scaled by the beach slope and incident wave conditions. The wave directional spread
strongly influences the exchange velocity time due to its role in surfzone vorticity generation.

The transient rip current exchange velocity cross-shore decay lengthscale is up to $2.5 \times$ the surfzone width, indicating the importance of surfzone processes on the inner shelf. These scalings can be used to quantify the surfzone flushing time which also depends on wave directional spread. These scaling laws allow estimation of the transient rip current cross-shelf exchange velocity and comparison to other inner-shelf exchange processes such as shoaling internal waves, shelf eddies, or wind-driven circulation. For example, transient rip current exchange velocity can be stronger than wave-induced Stokes’ drift exchange velocity up to six surfzone widths from shore well onto the inner-shelf.

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Figure 1. (a) Aerial photograph of an alongshore uniform beach during a shoreline dye release [Hally-Rosendahl et al., 2014]. Dye is ejected from the surfzone to the inner shelf in plumes associated with transient rip currents. (b - e) Results from a funwaveC simulation with $\beta = 0.04$ and incident $H_s = 1.1$ m, $T_p = 14$ s, $\sigma_\theta = 10^\circ$. (b) Snapshot of vertical vorticity ($\omega$) and current vectors for a subset of the model domain as a function of cross- ($x$) and alongshore ($y$) coordinate for a well-developed transient rip current exiting the surfzone. (c) Significant wave height $H_s$, (d) time-mean transient rip current driven exchange $U_{ex}$, and (e) planar bathymetry $h$ versus $x$ with shoreline at $x = 0$ m and the vertical dashed black line divides the surfzone and inner-shelf ($x = L_{sz}$). The Gaussian exchange velocity parameters ($U_{ex0}$, $x_o$, $L_d$) defined in (2) are noted in panel d.
Figure 2. (a) Cross-shore profiles of transient rip current driven exchange velocity $U_{ex}$, for 120 model simulations. (b) $U_{ex}$ profiles nondimensionalized by the three best-fit Gaussian parameters ($U_{ex0}$, $x_o$, $L_d$). (c - e), Nondimensional scale dependencies for each fit parameter; (c) the maximum strength of the offshore flow ($U_{ex0}$), (d) the cross-shore location of the maximum ($x_o$), and (e) the cross-shore decay lengthscale ($L_d$).
Figure 3. Nondimensional surfzone flushing time $T_f^r (g/h_b)^{1/2} \beta$ versus breakpoint wave directional spread $\sigma_{\theta_b}$. Red dashed curve shows $\sigma_{\theta_b}^{-1}$ dependence.
Figure 4. (a) Transient rip current \( U_{ex}^t \) (blue) and Stokes’ Drift \( U_{ex}^{st} \) (red) exchange velocities versus \( x \) for the simulation shown in Figure 1. Note the logarithmic vertical axis. The location \( X_I = 353 \) m where \( U_{ex}^t = U_{ex}^{st} \) (yellow circle, gray line) is 3.5 times the surfzone width \( L_{sz} = 97 \) m, black dashed line). (b) Nondimensional intersection location \( X_I/L_{sz} \) versus breaking Irribarren number \( Irb = \beta/\beta_b^{1/2} \) for all simulations. A value of \( X_I/L_{sz} = 2 \) indicates that transient rip current driven exchange is greater than Stokes’ drift driven exchange up to two surfzone widths from the shoreline.