Inhomogeneous Turbulent Dispersion across the Nearshore

Induced by Surfzone Eddies

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ABSTRACT

In various oceanic regions, drifter-derived diffusivities reach a temporal maximum and subsequently decrease. Often, these are regions of inhomogeneous eddies, however, the effect of inhomogeneous turbulence on dispersion is poorly understood. The nearshore region (spanning the surfzone to the inner-shelf) also has strong cross-shore inhomogeneous turbulence. Nearshore Lagrangian statistics are estimated from drifter trajectories simulated with a wave-resolving Boussinesq model of random, normally-incident, and directionally spread waves. The eddy field cross-shore inhomogeneity affects both the mean cross-shore drift, and cross- and alongshore diffusivities. Short-time diffusivities are locally ballistic and the mean drift is toward the eddy velocity variance maximum. The diffusivities reach a maximum and subsequently decrease, i.e., are subdiffusive. The diffusivity maximum and time-to-maximum are parameterized taking into account the eddy field inhomogeneity. At long-times, the cross- and alongshore diffusivities scale as $t^{-1/2}$ and $t^{-1/4}$, respectively, which is related to the offshore decay of the eddy intensity. A diffusion equation, with a space-dependent Fickian diffusivity related to the eddy velocity variance, reproduced the short, intermediate, and long-time behavior of the mean drift and cross-shore diffusivity. The small Middleton parameter, indicating fixed float dispersion, suggests the Eulerian time-scale can parameterize the Lagrangian time-scale in this region. Although this simulation had no mean currents, and thus no shear dispersion or mixing suppression, inhomogeneous turbulence effects may be relevant in other regions such as the ACC and Western Boundary Current extensions.
1. Introduction

The effect of turbulence on scalar mixing is a fundamental problem across geophysical fluid dynamics. For example, the meridional overturning circulation depends on both small scale \(O(10^{-2} \text{ m})\) vertical mixing that influences stratification (e.g., Wunch and Ferrari 2004), and large-scale (10–100 km) horizontal mixing, induced by mesoscale eddies, is critical to the meridional heat flux (e.g., Marshall et al. 2006). Turbulent mixing also affects ocean anthropogenic contaminant evolution on both large (basin) length-scales and multi-year time-scales, such as radiation levels from the Fukushima nuclear disaster (Nakano and Povinec 2012), and on small scales such as coastal wastewater plumes on \(O(1 \text{ km})\) length-scales and hour time-scales (e.g., Grant et al. 2005). Close to shore within the surfzone (region of depth-limited wave breaking), horizontal turbulence rapidly mixes tracer on \(O(10 \text{ m})\) length and \(O(100 \text{ s})\) time scales (Spydell and Feddersen 2009) and is critical to the exchange of heat, material, and passive biology between the surfzone and the inner-shelf (e.g., Hally-Rosendahl et al. 2014; Morgan et al. 2018), the region denoted here as the nearshore.

Our understanding of turbulent scalar mixing is based mainly on idealized turbulence that is isotropic, homogeneous, and stationary. In this idealized setting, the eddy diffusivity \(K\) quantifies turbulent scalar mixing and is defined as the ensemble mean spreading rate

\[
K(t) = \frac{1}{2} \frac{d}{dt} D^2(t),
\]  

(1)

where \(D^2(t)\) is the variance of drifter positions, or the second moment of tracer concentration, at time \(t\) after release. The eddy diffusivity has two important limits (Taylor 1922)

\[
K(t) = \begin{cases} 
U^2t & \text{for } t \ll T_L \\
U^2T_L & \text{for } t \gg T_L,
\end{cases}
\]  

(2a) (2b)
denoted ballistic and Brownian dispersion, respectively. Here, $U^2$ is the eddy velocity variance and $T_L$ is the Lagrangian time-scale: the decorrelation time of Lagrangian velocities. For long-time Brownian dispersion, the asymptotic diffusivity is $K^\infty \equiv U^2 T_L$. Although $K^\infty$ is fundamentally a Lagrangian flow property due to $T_L$, for isotropic and homogeneous turbulence, it can be related to Eulerian eddy time- and length-scales (Middleton 1985).

In many instances, particularly near boundaries, turbulent inhomogeneities must be accounted for when estimating turbulent transport and dispersion. Two examples where boundary-induced inhomogeneity affects dispersion are the atmospheric boundary layer (ABL) and turbulent channel flow. For the ABL, the mean flow, eddy velocity variance, and turbulent length-scales depend on the vertical $z$, such that vertical variations in the eddy velocity and Lagrangian time-scale must be accounted for to accurately model scalar dispersion (Wilson et al. 1981). Due to inhomogeneous turbulence and boundary effects, rather than approaching a constant $K^\infty$, the ABL vertical eddy diffusivity $K_z(t)$ decreases in time after reaching a maximum (Dosio et al. 2005) indicative of subdiffusive dispersion. A similar time-dependence is observed for the cross-channel diffusivity for turbulent channel flow (Choi et al. 2004).

The nearshore region, spanning the surfzone and inner-shelf, have dramatically different dynamical regimes resulting in cross-shore inhomogeneous turbulence. The surfzone horizontal eddy field (vertical vorticity) is stochastically forced by wave-breaking resulting in a range of eddy time- and length-scales (Spydell and Feddersen 2009; Feddersen 2014). In contrast, the inner-shelf is, by definition, not breaking-wave eddy forced. Also inducing inhomogeneity (and anisotropy), the surfzone has a shoreline boundary, and the depth varies in the cross-shore inducing vortex stretching (e.g., Arthur 1962). These effects lead to a complex surfzone and inner-shelf eddy field (e.g., Suanda and Feddersen 2015) where horizontal eddy magnitudes can $O(1)$ vary over
the width of the surfzone $L_{sz}$ (Feddersen et al. 2011). As the surfzone width $L_{sz}$ is similar to the length-scale of the energy containing eddies, the assumption of homogeneous turbulence is strongly violated.

Despite the inhomogeneities, surfzone diffusivities have generally been estimated for the entire surfzone – mostly due to the limitations of the observations – tacitly assuming cross-shore homogeneity. Dye tracer has been used to estimate surfzone cross-shore diffusivity from dye moments integrated across the surfzone (Clark et al. 2010). Drifter-derived time-dependent diffusivities $K_x(t)$ have been calculated for $\leq 10^3$ s with estimated Lagrangian time-scales $T_L$ of 100-200 s (Spydell et al. 2009a; Brown et al. 2009; Spydell and Feddersen 2012b). Drifter-derived alongshore ($y$) diffusivities $K_y(t)$ increase with the strength of the mean alongshore current $V$ (Spydell and Feddersen 2012b) due to shear dispersion that takes into account $T_L$ (Spydell and Feddersen 2012a). In Spydell et al. (2007) and Brown et al. (2009), separate diffusivities were calculated in the regions within and seaward of the surfzone. However, the binning was too coarse with insufficient statistical reliability to illuminate inhomogeneous turbulence effects. In Spydell and Feddersen (2012b), explicit shoreline effects were considered, but otherwise, the turbulence was assumed homogeneous. The relationship between surfzone-averaged Lagrangian and Eulerian time-scales was explored in Spydell et al. (2014). In these previous studies, cross-shore homogeneity is assumed because the number (between 10–30), and trajectory-length ($\leq 10^3$ s), of drifter observations were insufficient to estimate the cross-shore dependence of $K(t)$ or other Lagrangian statistics with any accuracy (Spydell et al. 2007; Brown et al. 2009; Spydell et al. 2009a). Furthermore, because the diffusivity estimate error grows in time (e.g., Davis 1991), whether the surfzone cross-shore diffusivity approaches a constant Brownian regime is unclear (e.g., Spydell and Feddersen 2012b).
Thus, several important questions remain regarding diffusion in the inhomogeneous turbulence spanning the surfzone to inner-shelf. One, why is there a diffusivity maxima with subsequent decrease and how can it be scaled? Two, how does inhomogeneous turbulence affect long-time dispersion and exchange from the surfzone onto the inner-shelf? Here, dispersion spanning the inhomogeneous turbulence of the surfzone to inner-shelf is simulated using modeled trajectories 10× longer than previously considered and many more trajectories than observationally possible. This allows the cross-shore structure of surfzone to inner-shelf particle transport and dispersion to be quantified in detail.

Drifters are tracked in an idealized but realistic nearshore simulation with random normally-incident but directionally spread waves using the wave-resolving funwaveC model, from which time- and release-location dependent Lagrangian statistics are calculated (Section 2). The simulation had no mean flows but strongly cross-shore inhomogeneous turbulence. The time- and cross-shore release location dependence of the modeled cross-shore mean drift and cross- and alongshore diffusivities are presented in Section 3. The short-time, intermediate-time diffusivity maxima, and long-time behavior of the mean drift and diffusivities are scaled in Section 4. The ability of a Fickian diffusion equation, with a time- and cross-shore dependent Fickian diffusivity, to represent the modeled mean drift and cross-shore diffusivity is examined in Section 5. In the Discussion (Section 6), Lagrangian and Eulerian statistics are compared and the applicability of the results to other regions with inhomogeneous eddy fields is discussed. Results are summarized in Section 7.

2. Model Setup and Definitions
a. The Model

The surfzone Eulerian and Lagrangian statistics analyzed here are based on a single simulation of the Boussinesq model *funwaveC* (Feddersen et al. 2011). Boussinesq models have been validated for laboratory waves (Shi et al. 2012), surfzone field observations (Feddersen et al. 2011), observed surfzone drifters (Spydell and Feddersen 2009), and observed surfzone dye plumes (Clark et al. 2011). The model solves the finite difference approximations of the Boussinesq mass and momentum equations with nonlinear and dispersive effects (Nwogu 1993) with biharmonic friction with hyperviscosity of $\nu_{bh} = 0.3 \, \text{m}^4 \, \text{s}^{-1}$ and quadratic bottom friction with a drag coefficient $c_d = 0.0023$ appropriate for the surfzone (Feddersen et al. 2011). Depth-limited wave breaking is parameterized with an eddy viscosity (Kennedy et al. 2000) and wave runup is implemented with the “thin-layer” method (Salmon 2002). An offshore sponge layer ensures outgoing wave energy is not reflected.

The alongshore uniform idealized planar beach has bathymetry $h(x) = -sx$, where $x$ is the cross-shore coordinate increasing onshore with shoreline at $x = 0$, and slope $s = 0.03$ typical of many beaches (Fig. 1a). At a depth of $h = 9 \, \text{m}$ (at $x = -300 \, \text{m}$), the bathymetry is constant farther offshore ($x < -300 \, \text{m}$). The cross- and alongshore model domain size is $(L_x, L_y) = (627, 1600) \, \text{m}$, with grid resolution $(\Delta x, \Delta y) = (1, 1.33) \, \text{m}$, and is periodic in the alongshore direction. In $h = 9 \, \text{m}$ water depth (at $x = -444 \, \text{m}$), the source function method (Wei et al. 1999; Suanda et al. 2016) generates random directionally-spread waves with a Pierson-Moskovitz spectrum, significant wave height $H_s = 0.8 \, \text{m}$, peak period $T_p = 8 \, \text{s}$, and mean-normal incidence $\bar{\theta} = 0$. The $H_s(x)$ increases as waves shoal and then decreases through the surfzone due to breaking wave dissipation (Fig. 1b). The cross-shore location of maximum $H_s(x)$, defines an offshore limit of the surfzone $x_{sz} = -90 \, \text{m}$. 

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At $x_{sz}$, the directional spread (Kuik et al. 1988) is $\sigma_\theta = 10^\circ$. Model data is output at 1 Hz.

Surfzone eddies possessing vertical vorticity, forced by finite crest-length wave breaking (Peregryn 1998; Clark et al. 2012), are responsible for surfzone dispersion (Spydell and Feddersen 2009). As such, model velocities $u_m = (u_m, v_m)$ are separated into rotational and irrotational components,

$$u_m = \nabla \times \psi k + \nabla \phi.$$  

(3)

The rotational, or eddy velocities, $u$ are then found from the stream function $\psi$

$$u = \nabla \times \psi k$$  

(4)

which is obtained from the vorticity, $\nabla^2 \psi = \zeta$, where $\zeta = (\nabla \times u_m) \cdot k$. For $>100$ s, irrotational velocities ($\nabla \phi$) do not contribute to surfzone particle dispersion (Spydell and Feddersen 2009).

For the normally incident waves simulated here, the time-mean cross- and alongshore eddy velocities ($\langle u \rangle$, $\langle v \rangle$) are zero. The cross-shore dependent Eulerian cross- and alongshore eddy velocity variances $(U^2(x), V^2(x))$ are

$$(U^2, V^2) = \langle (u^2), (v^2) \rangle.$$  

where $\langle \rangle$ denotes time- and alongshore-averaging.

From mid-surfzone to offshore of the surfzone, cross-$U^2(x)$ and alongshore $V^2(x)$ eddy velocity variances are similar and vary from about 0.015 m$^2$ s$^{-2}$ at $x = -50$ m to 0.001 m$^2$ s$^{-2}$ at $x = -140$ m (Fig. 1c,d). Onshore of $x = -50$ m, $V^2$ is larger than $U^2$, due to the shoreline limiting cross-shore velocities, and both decrease. The maximum cross-shore ($U_{mx}^2$) and alongshore eddy velocity variance ($V_{mx}^2$) occur at $x_{U_{mx}} = -40$ m and $x_{V_{mx}} = -20$ m, respectively (Fig. 1c,d).

An analytic expressions for $U^2(x)$ will be used in later analysis. The analytic $U_{an}^2(x)$ is

$$U_{an}^2(x) = \frac{U_{mx}^2}{1 + [(x - x_{U_{mx}})/Lx]^2}.$$  

(5)

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with $U_{\text{mix}}^2 = 0.015 \text{ m}^2 \text{s}^{-2}$, and $l_x = 38 \text{ m}$ (approximately $|x_{U_{\text{max}}}|$) is the cross-shore length-scale over which the turbulence varies. This analytic form $U_{\text{mix}}^2(x)$ generally reproduces the $U^2(x)$ (dashed red curve in Fig. 1c), although it does not decay rapidly enough onshore of $x_{U_{\text{mix}}}$ and slightly over-estimates $U^2$ offshore of $x_{sz}$. The $V^2(x)$ is also generally well-represented by the same analytic form but with different parameters.

**b. Drifter Simulations**

Virtual drifter tracks $X(t)$ are determined by advection from the eddy velocities $u$,

$$X(t) = X_0 + \int_0^t u(X(t'), t') \, dt'$$

where $X_0 = (X_0, Y_0)$ is the initial drifter position. A 4th order Runge-Kutta algorithm is used to integrate (6) with $u(x, y, t)$ fields sampled at 1 Hz and with $u(x, y, t')$ linearly interpolated to drifter positions $X(t')$. To insure stationarity, the model is spun up for 2000 s before virtual drifters are released between $-150 \leq x \leq 0$ and for all $y$ on a regular grid of 31 drifters by 100 drifters (corresponding to $\Delta x = 5 \text{ m}$ and $\Delta y = 16 \text{ m}$) in the cross- and alongshore directions, respectively. Every 250 s after the first release of 3100 drifters, 3100 additional drifters are released on the same regular grid. Each release is quasi-independent as the eddy Eulerian decorrelation time is $\approx 250 \text{ s}$, resulting in a total of 53 quasi-independent releases of 3100 drifters. The resulting drifter trajectories vary in length from 5000 s to $1.8 \times 10^4 \text{ s}$.

For one of the drifter realizations, the time evolution of drifter positions reveals a complex stirring field spanning the surfzone to the inner-shelf (Fig. 2). Drifters are initially released on a regular grid at $t = 0 \text{ s}$ (Fig. 2a). At $t = 100 \text{ s}$ (Fig. 2b), drifters released shoreward of $x \approx x_{U_{\text{mix}}}$ (red dots) have been noticeably stirred by the surfzone eddies whereas inner-shelf released drifters
(blue dots) have barely moved. At \( t = 250 \) s (Fig. 2c), drifters released near the shoreline (red dots), sampling strong eddies (vorticity, see \( y \approx 325, 550, 750 \) m), have been strongly stirred to the surfzone boundary (gray dashed line, \( x_{sz} = -90 \) m) whereas inner-shelf released drifters are only somewhat distorted from their release pattern. At \( t = 1000 \) s, strong stirring is evident for surfzone-released drifters (red dots are seaward of yellow dots in Fig. 2d), but inner-shelf released drifters (blue dots) have only been weakly stirred. At \( t = 4000 \) s (Fig. 2e), surfzone released drifters are fully-stirred within the surfzone, but only about half of the inner-shelf released drifters (blue dots) have entered the surfzone. Lagrangian statistics based on these trajectories will quantify the cross-shore variability of the dispersion evident in Fig. 2.

**c. Definition of Lagrangian Statistics**

Lagrangians statistics that quantify scalar transport and mixing are estimated from the drifter trajectories. The mean cross-shore trajectory (i.e., center of mass location) for drifters released from \( X_0 \) is given by

\[
\bar{X}(t, X_0) = \langle X(t, X_0) \rangle,
\]

where \( t \) denotes time from release, the mean \( \langle \cdot \rangle \) is over all 100 drifters released from \( X_0 \) and over the 53 quasi-independent realizations. The mean drift velocity \( \bar{U} \) is

\[
\bar{U}(t, X_0) = \frac{d\bar{X}(t, X_0)}{dt},
\]

and equivalent to the average velocity over all drifters released from \( X_0 \). The cross-shore dispersion is defined as

\[
D_{xz}^2(t, X_0) = \langle [X(t, X_0)]^2 \rangle - [\bar{X}(t, X_0)]^2,
\]
and represents the variance of drifter positions about their mean position $\bar{X}(t, X_0)$ for all drifters released from $X_0$. The Lagrangian velocity variance $U_{L}^2$ is defined similarly

$$U_{L}^2(t, X_0) = \langle [U(t, X_0)]^2 \rangle - \langle [\bar{U}(t, X_0)]^2 \rangle,$$  \hspace{1cm} (10)$$

and is equal to the Eulerian velocity variance only initially, $U_{L}^2(t = 0, X_0) = U^2(X_0)$, before drifters have dispersed from $X_0$. For homogeneous turbulence, $U_{L}^2$ does not depend on $t$ or $X_0$ and is equal to the Eulerian velocity variance $U^2$. The cross-shore eddy diffusivity $K_x(t, X_0)$ is related to the dispersion via

$$K_x(t, X_0) = \frac{1}{2} \frac{d}{dt} D_x^2(t, X_0).$$ \hspace{1cm} (11)$$

and for isotropic and homogeneous turbulence is equivalent to (1). The alongshore dispersion $D_y^2(t, X_0)$, Lagrangian velocity variance $V_{L}^2(t, X_0)$, and diffusivity $K_y(t, X_0)$ are similarly defined. Time derivatives are estimated as simple Euler forward differences. Note, that with the releases staggered by 250 s, the number of realizations used in the mean (and thus degrees of freedom) decreases at $t > 5000$ s. Because estimates of $K$ using (11) are notoriously noisy (e.g., Spydell et al. 2009b; LaCasce et al. 2014), high frequency noise in $K_x(t, X_0)$ and $K_y(t, X_0)$ (and $\bar{U}(t)$) is removed by time-smoothing $K_i(t)$ (and $\bar{U}(t)$) using a box car filter where filter length increases linearly with time, accounting for the long-time decrease in degrees of freedom in the mean.

### 3. Lagrangian statistics

The release-location dependent mean cross-shore trajectory $\bar{X}(t, X_0)$ and drift velocity $\bar{U}(t, X_0)$ are dominated by a few features (Fig. 3a,b). The first is the initial drift towards the location of the $U^2$ maximum $x_{U_{\text{max}}} = -40$ m (gray line in Fig. 3a), that is shoreward ($\bar{U} > 0$) for $x < x_{U_{\text{max}}}$ m and...
offshore ($\bar{U} < 0$) for $x > x_{U\text{max}}$ m (Fig. 3b). Onshore of $x_{U\text{max}}$, the $\bar{U}$ decreases rapidly until reaching a relatively large minimum of $-0.02$ m s$^{-1}$ to $-0.035$ m s$^{-1}$ at 100-300 s (maximum offshore velocity, open circles on red curves in Fig. 3b), with generally deeper and earlier minimum for $X_0$ closer to the shore. Offshore of $x_{U\text{max}}$, the $\bar{U}$ increases to a maximum ($0.005–0.021$ m s$^{-1}$) that generally is weaker and occurs later for larger $X_0$. Subsequently, the drift velocity magnitude ($|\bar{U}|$) decays with time. For $-125 < X_0 < x_{U\text{max}}$ m, the onshore $\bar{U}$ transitions to offshore drift, i.e., $\bar{U}(t)$ passes through 0, with the transition time occurring later for more offshore $X_0$ (Fig. 3a). For example, just offshore of $x_{U\text{max}}$ ($X_0 = -55$ m, yellow curve Fig. 3a,b), the transition occurs at $\approx 400$ s, whereas just offshore of the surfzone ($X_0 = -95$ m, cyan), the transition occurs at $\approx 3000$ s. At long times, the mean drift velocities $\bar{U}$ collapse for all $X_0$.

The cross- and alongshore Lagrangian velocity variances ($U^2_L(t, X_0), V^2_L(t, X_0)$) are time dependent (Fig. 3c,d) due to drifters sampling varying eddy velocities as they disperse from $X_0$. At very short times ($t \leq 20$ s), the cross- and alongshore Lagrangian and Eulerian velocity variance are very similar $U^2_L(t, X_0) \approx U^2_i(X_0)$ as the drifters have not yet dispersed and the $U^2_L$ are essentially constant in time. The time when $U^2_L$ and $V^2_L$ become time-varying depends on the distance between $X_0$ and $x_{U\text{max}}$ or $x_{V\text{max}}$ (Fig. 3c,d). For example, for a near-shoreline ($X_0 = 5$ m) release, $U^2_L$ increases at approximately 30 s, reaching a maximum at about 600 s that is $\approx 3\times$ the initial value (reddest curve, Fig. 3c). For the farthest offshore released drifters ($X_0 = -145$ m), $U^2_L$ and $V^2_L$ start increasing at about 1000 s to values $\approx 6\times$ their initial values. Regardless of the release location $X_0$, both $U^2_L(t, X_0)$ and $V^2_L(t, X_0)$ eventually increase from their respective initial $U^2_i(X_0)$. The precise evolution of $U^2_L$ and $V^2_L$ differ because the Eulerian cross- $U(X_0)$ and alongshore $V(X_0)$ eddy velocities have different cross-shore structure (Fig. 1c). At long times, $U^2_L$ (as well as $V^2_L$) converge for all $X_0$ and have an approximate asymptotic time-dependence of $t^{-1/4}$ (dashed line in
The cross-shore diffusivity and alongshore diffusivities ($K_x, K_y$) each have qualitatively similar time-dependence across all $X_0$ (Fig. 3e,f). Initially ($t < 50$ s), the cross-shore diffusivity is ballistic for all $X_0$ with $K_x \propto t$ (Fig. 3e), as expected for homogenous turbulence. Thereafter, the cross-shore diffusivity $K_x$ reaches a maximum value $K_x$ at time $T_x$, and subsequently decreases indicating a subdiffusive (as opposed to Brownian) regime (Fig. 3e). The $K_x$ maximum and time to maximum ($K_x, T_x$) range from ($1 \text{ m}^2\text{s}^{-1}, 150$ s) for $X_0 = x_{U_{\text{m}x}}$, and ($0.45 \text{ m}^2\text{s}^{-1}, 2000$ s) for $X_0$ offshore of surfzone for a factor $2.2 \times$ and $13 \times$ variation in $K_x$ and $T_x$, respectively. In general, $T_x$ and $K_x$ are proportional to and inversely proportional to the distance from release to $x_{U_{\text{m}x}}$ ($|X_0 - x_{U_{\text{m}x}}|$). For times longer than $T_x$, the cross-shore diffusivity $K_x$ is subdiffusive, scaling as $K_x \propto t^{-1/2}$ (dashed line in Fig. 3e). At even longer times, $K_x$ becomes independent of $X_0$ (note collapse of red–cyan curves for $t > 1000$ s), with the time of lost $X_0$-dependence also depending on $|X_0 - x_{U_{\text{m}x}}|$. For the most offshore released drifters ($X_0 = -145$ m), the $X_0$ dependence is lost at $t > 6000$ s.

The alongshore diffusivity $K_y$ is generally larger than $K_x$ but has qualitatively similar time-dependence (Fig. 3f). Like $K_x$, the initial $K_y$ time-dependence is ballistic. For offshore released drifters ($X_0 < -120$ m), after the initial ballistic growth, $K_y$ growth noticeably slows but then subsequently accelerates due to the strong time-varying $V_L^2(t)$ (Fig. 3d). Subsequently, the alongshore diffusivity $K_y(t, X_0)$ also reaches a maximum $K_y$ at time $T_y$ (Fig. 3f), ranging from ($K_y, T_y = 4 \text{ m}^2\text{s}^{-1}, 210$ s) for $X_0 = x_{V_{\text{m}x}}$ and ($K_y, T_y = 1 \text{ m}^2\text{s}^{-1}, 10^4$ s) for offshore $X_0$ (dark blue in Fig. 3f). Similarly, $T_y$ and $K_y$ are proportional to and inversely proportional to the release distance from $x_{V_{\text{m}x}}$ ($|X_0 - x_{V_{\text{m}x}}|$). At long times ($> T_y$), $K_y$ decreases subdiffusively $K_y \propto t^{-1/4}$ (dashed lines, Fig. 3f), but decreases more slowly than $K_x$ with its $t^{-1/2}$ scaling. At these long times, $K_y$
also becomes independent of $X_0$ and the $K_y$ curves collapse.

4. Scaling the Lagrangian statistics

a. Scaling the short-time diffusivity and mean drift

1) Diffusivity

For all $X_0$, the initial ($t < 50$ s) diffusivity growth $K_i \propto t$ (Fig. 3e,f) indicates ballistic dispersion (2a). For homogeneous turbulence, $K/U^2 = t$ for ballistic scaling. Similarly, $K_x$ and $K_y$ here are scaled by the eddy velocity variance $(U^2, V^2)$ at the release location $X_0$ (Fig. 1c,d). For $t < 50$ s, the time-dependence of $K_x(t, X_0)/U^2(X_0)$ (Fig. 4a) and $K_y(t, X_0)/V^2(X_0)$ (Fig. 4b) scales as $t$ and collapses for all $X_0$. This indicates that the short-time drifter dispersion is ballistic with the local (release location) eddy velocity variance. The ballistic scaling breaks down earlier for $K_x$ than for $K_y$ (compare Fig. 4a,b). The $K_x$ ballistic scaling breaks down sooner for $X_0$ near $x_{U_{\text{max}}}$ (i.e., small $|X_0 - x_{U_{\text{max}}}|$). For example, at $X_0 = x_{U_{\text{max}}}$ (orange curve in Fig. 4a), the ballistic scaling is strictly applicable to about $t = 50$ s, whereas for $X_0 = -145$ (far from $x_{U_{\text{max}}}$), it is applicable to $\approx 100$ s (blue curve in Fig. 4a), highlighting the importance of the distance $|X_0 - x_{U_{\text{max}}}|$. For $K_y$, a clear $X_0$ dependence for when the ballistic scaling breaks down is not evident (Fig. 4b).

2) Mean drift

The short-time mean drift velocity $\bar{U}$ is nonzero and directed towards $x_{U_{\text{max}}}$ (Fig. 3b). For zero
mean flow and homogeneous turbulence, the mean drift $\bar{U}$ is zero, whereas if the turbulence is inhomogeneous, gradients in the eddy diffusivity induce a non-zero mean drift velocity $\bar{U}$ (e.g., Davis 1991). For short times, before drifters have dispersed far from $X_0$, this relationship is

$$\bar{U}(t, X_0) = \frac{d}{dX_0} K_x(t, X_0).$$ \hspace{1cm} (12)

Because the short-time ($t < 50$ s) $K_x$ is ballistic for all $X_0$ (Fig. 4), (12) becomes

$$\bar{U}(t, X_0) = \frac{dU^2(X_0)}{dX_0} t, \hspace{1cm} (13)$$

and $\bar{U}$ grows linearly in time with constant mean drift acceleration of

$$\frac{d}{dt} \bar{U}(t, X_0) = \frac{d}{dX_0} U^2(X_0). \hspace{1cm} (14)$$

In the ballistic regime at $t = 10$ s, (14) accurately gives the mean drift acceleration $d\bar{U}/dt$ (Fig. 5) with very high skill. The $\bar{U} \propto t$ scaling is consistent with the $\bar{U}$ time evolution (Fig. 3b). Because $U^2$ and ballistic $K_x$ are maximum at $x_{U\text{max}}$, the short-time drift is towards $x_{U\text{max}}$ (Fig. 3a,b). At these short times, the presence of the shoreline does not impact the mean drift acceleration. At $t = 100$ s, (12) breaks down for $X_0$ near $x_{U\text{max}}$ (not shown) because $K_x$ is no longer ballistic (i.e., orange curve in Fig. 4a). In contrast, for $X_0$ far from $x_{U\text{max}}$, (12) still holds at $t = 100$ s because $K_x$ is still largely ballistic (i.e., blue curves in Fig. 4a).

**Fig. 5**

\[ \text{b. Scaling the maximum and timing of the diffusivity } K \text{ and } T \]

1) **The diffusivity maximum $K_x$ and $K_y$**
The magnitude of the diffusivity maxima $K_x$ and $K_y$ can vary significantly (Fig. 3e,f). Over all release locations, $K_x(X_0)$ varies by a factor of $\approx 2$, with maximum near $x_{U_{\text{max}}}$ (the location of $U^2$ maximum) and minimum offshore (Fig. 3e,6a). The $K_y(X_0)$ varies similarly with release location, with maximum near $x_{V_{\text{max}}}$ (Fig. 3f,6b). Although the release location velocity variance ($U^2(X_0)$ and $V^2(X_0)$) determines the initial ballistic dispersion, it does not collapse $K_x$ or $K_y$ for longer times (Fig. 4a,b). As drifters cross-shore disperse from $X_0$, they sample regions of differing eddy velocities resulting in a time-dependent Lagrangian velocity variance $U^2_{L_i}$ (Fig. 3c,d). Here, an “effective” eddy velocity variance is sought that can scale $K_x$ and $K_y$ in a form analogous to Brownian diffusion (2b). Initially, drifters drift from $X_0$ towards $x_{U_{\text{max}}}$ (Fig. 3a), hence, an effective Eulerian cross-shore eddy velocity variance $\bar{U}^2(X_0)$ is defined as the cross-shore average of $U^2(x)$ from $X_0$ to $x_{U_{\text{max}}}$,

$$\bar{U}^2(X_0) = \frac{1}{X_0 - x_{U_{\text{max}}}} \int_{X_0}^{x_{U_{\text{max}}}} U^2(x') \, dx'.$$  

(15)

The effective alongshore eddy velocity variance $\bar{V}^2(X_0)$ is defined similarly, but averaged between $X_0$ and the location of the maximum alongshore eddy velocity $x_{V_{\text{max}}}$

$$\bar{V}^2(X_0) = \frac{1}{X_0 - x_{V_{\text{max}}}} \int_{X_0}^{x_{V_{\text{max}}}} V^2(x') \, dx'.$$  

(16)

The parameterization for $\mathcal{K}$ is then expressed as

$$K_x(X_0) = \bar{U}^2(X_0) \bar{T}_{Lx}$$  

(17)

$$K_y(X_0) = \bar{V}^2(X_0) \bar{T}_{Ly}$$  

(18)

where the time-scales $\bar{T}_{Lx}$ and $\bar{T}_{Ly}$ are spatially-uniform. These time-scales are denoted “effective” cross- and alongshore Lagrangian time-scales as they are associated with the maximum diffusivity in analogy with $T_L$ being associated with $K^\infty$ in homogeneous, isotropic turbulence (Taylor 1922). Note that $\bar{T}_{Lx}$ and $\bar{T}_{Ly}$ are not decorrelation time-scales as $T_L$ is understood to be.
With this simple effective cross-shore velocity variance $\bar{U}^2$, the parameterized $K_x(X_0)$ (17) represents well the drifter-derived $K_x(X_0)$ with a constant best-fit $\bar{T}_{Lx} = 66$ s (Fig. 6a). Similarly, in the alongshore, the parameterized $K_y(X_0)$ (18) represents well the drifter-derived $K_y(X_0)$ for $X_0 < x_{U_{\text{mix}}}$ with a constant best-fit $\bar{T}_{Ly} = 146$ s (Fig. 6b). However, in the inner-surfzone for $X_0 > x_{U_{\text{mix}}}$, this $K_y$ scaling breaks down with 30-50% errors. In this near-shoreline region, a smaller $\bar{T}_{Ly}$ is required in this parameterization analogous to how atmospheric boundary layer, and turbulent channel flow, Lagrangian time-scales decrease towards the boundary (Wilson and Sawford 1996; Choi et al. 2004).

2) THE TIME TO THE DIFFUSIVITY MAXIMUM $T_x$ AND $T_y$

The time of the diffusivity maximum $T_x(X_0)$ and $T_y(X_0)$ is hypothesized to be proportional to the time for drifters to disperse across the width of the strong eddying region, (across $U^2(x)$), approximately the surfzone width (Figure 1c). This time is proportional to a length-scale $\mathcal{L}$ divided by a velocity scale. Because $K_x$ is approximately ballistic up to $T_x$ (Fig. 3e), the release location eddy velocity $U(X_0)$ is used as a velocity scale. The strong $U^2$ region has max at $x_{U_{\text{mix}}} = -40$ m with width $\approx 2|x_{U_{\text{mix}}}|$. For offshore release $X_0 < x_{U_{\text{mix}}}$, drifters must disperse from the release location to the shoreline resulting in the length-scale $\mathcal{L}_x = |X_0|$. For release onshore of $x_{U_{\text{mix}}}$, drifters must disperse from $X_0$ to $2x_{U_{\text{mix}}}$, so that $\mathcal{L}_x = -2x_{U_{\text{mix}}} + X_0$. For all $X_0$, the length-scale is $\mathcal{L}_x = |X_0 - x_{U_{\text{mix}}}| + |x_{U_{\text{mix}}}|$, and $T_x$ is parameterized as

$$T_x(X_0) = \frac{A_1}{U(X_0)} (|X_0 - x_{U_{\text{mix}}}| + |x_{U_{\text{mix}}}|), \quad (19)$$

where $A_1$ is a non-dimensional constant. This parameterization (19) reproduces well the drifter-derived $T_x(X_0)$ with an $O(1)$ best-fit $A_1 = 0.35$ (compare black and red curves in Fig. 6c), suggest-
ing that once drifters quasi-ballistically disperse across the strong eddying region, the maximum
diffusivity is reached.

Here, $\tau_y$ is similarly related to time for cross-shore drifter dispersion through the strong $V^2(x)$
region, with similar parameterization. The cross-shore velocity scale is again $U(X_0)$, as drifters
must disperse across the strong $V^2$ region with peak at $x_{V_{mx}}$ and half-width of $x_{V_{mx}}$ (Figure 1c).
This gives a length-scale of $L_y = |X_0 - x_{V_{mx}}| + |x_{V_{mx}}|$, and $\tau_y(X_0)$ is then parameterized as

$$\tau_y(X_0) = \frac{A_2}{U(X_0)} \left( |X_0 - x_{V_{mx}}| + |x_{V_{mx}}| \right).$$  (20)

This parameterization reproduces reasonably well the drifter-derived $\tau_y(X_0)$ structure (Fig. 6d).

The $O(1)$ best-fit $A_2 = 2.5$ is about $\times 8$ larger than $A_1$ as $\tau_y$ is generally larger than $\tau_x$. For $X_0$
offshore of $x_{sz}$, the parameterization results in times too small, likely because the assumption of
cross-shore locally ballistic dispersion has already broken down (Fig. 3e).

c. Scaling the long-time Lagrangian statistics

1) MEAN DRIFT

At long times, after drifters have sufficiently dispersed so that the shoreline is felt, the shore-
line limits onshore displacements imparting a mean negative drift velocity $\bar{U}$ (e.g., Spydell and
Feddersen 2012b). In general, the time when the shoreline affects $\bar{U}$, that is the maximum offshore
drift velocity (open circles in Fig. 3b), increases with offshore $X_0$. Thereafter, long-time offshore
drift collapses for most $X_0$, except for the farthest offshore $X_0$. For homogeneous turbulence with
Brownian dispersion ($D_x^2 \propto t$), but with a reflecting shoreline, the long-time cross-shore trajectory
\(\bar{X}\) scales as (Spydell and Feddersen 2012b)

\[
|\bar{X}(t)| \propto [D_x^2(t)]^{1/2} \propto t^{1/2},
\]

(21)

resulting in an offshore directed mean trajectory. As the long-time \(D_x^2\) is related to \(K_x\) through (11), the long-time subdiffusive \(K_x \propto t^{-1/2}\) found here corresponds to \(D_x^2 \propto t^{1/2}\) (not \(t^1\)). Applying the scaling (21) gives \(\bar{X} \propto t^{1/4}\) which matches \(\bar{X}\) for mid-surfzone release (compare dashed-black and yellow curve in 3a). Similarly, this scaling gives \(\bar{U} \propto t^{-3/4}\), matching the long-time offshore velocity (compared dashed-black and colors in Fig. 3b). Thus, the mean trajectory scaling \(\propto t^{1/4}\) is also applicable in inhomogeneous turbulence with a boundary, although \(\bar{X}\) moves offshore more slowly here than for homogeneous turbulence due to offshore decaying turbulence.

2) DIFFUSIVITIES

Here, the subdiffusive behavior of the long-time cross-shore \((K_x \sim t^{-1/2})\), and alongshore \((K_y \sim t^{-1/4})\), diffusivities (Figure 3e,f) are investigated. For the cross-shore diffusivity, the approach to scaling the long-time behavior is analogous to that for scale-dependent relative diffusivities (Richardson 1926; Spydell et al. 2007). At long times, drifters are cross-shore well-mixed within the surfzone, so that the magnitude of \(K_x\) is a result of drifter dispersing farther offshore as \(D_x\) (9) increases. At these long times, the local \(K_x\) is assumed Brownian (2b) with form \(K_x \propto U_{\text{an}}^2(x) T_L\), such that offshore of the surfzone, the analytic \(U_{\text{an}}^2 \sim x^{-2}\) (5). Here, \(T_L\) is assumed spatially uniform, analogous as was found for the effective \(\bar{T}_{Lx}\) in the \(K_x\) scaling (Fig. 6a). Thus, only the \(U^2\) inhomogeneity affects the diffusivity. For the local \(K_x\), \(D_x\) is assumed proportional to \(|x|\)
resulting in

\[ K_x \propto D_x^{-2}, \tag{22} \]

so that drifters feel weaker diffusivities the farther offshore they have dispersed. In contrast, \( K \sim D^{4/3} \) for turbulent relative dispersion (Richardson 1926). Using (22) in (11) results in a long-time dispersion scaling of \( D_x^2 \sim t^{1/2} \) and long-time subdiffusive \( K_x \sim t^{-1/2} \) with power-law scaling consistent with the long-time \( K_x \) for surfzone \( X_0 \) (Figure 3e). Thus, the subdiffusive power-law for \( K_x \) is set by the spatial decay of the eddy velocities \( \mathcal{U}^2 \) (explored further in Section 5c).

The long-time alongshore diffusivity scaling \( K_y \sim t^{-1/4} \) is now investigated. Unlike the crossshore dispersion which is uncoupled from the alongshore dispersion, the alongshore dispersion is coupled to the cross-shore dispersion because \( V^2 \) is a function of \( x \). Thus, \( K_y \) depends directly on the cross-shore distribution of drifters. For long-times, and assuming a constant alongshore Lagrangian time-scale \( T_{Ly} \), the local (if drifters did not spread in \( x \)) alongshore eddy diffusivity would be \( K_y(x) = V^2(x)T_{Ly} \). However, because drifters spread in \( x \), the eddy diffusivity \( K_y \) is the cross-shore position weighted average of \( V^2(x)T_{Ly} \), i.e.,

\[ K_y(t, X_0) = T_{Ly} \int V^2(x) p(x, t, X_0) \, dx, \tag{23} \]

where \( p(x, t, X_0) \) is the \( x \) distribution of drifters released from \( X_0 \) at time \( t \). For simplicity, drifters are assumed uniformly distributed from the shoreline to \( -D_x \), and \( V^2(x) \) has the functional form \( V_{an}^2(x) \) (Fig. 1d and Eq. (5)), thus,

\[ K_y(t) \approx \frac{V_0^2 T_{Ly}}{D_x(t)} \int_{-D_x(t)}^{0} \frac{1}{1 + [(x - x_0)/l_x]^2} \, dx \]

\[ \approx \frac{V_0^2 T_{Ly}}{D_x(t)} \frac{l_x}{D_x(t)} \left[ \tan^{-1} \left( \frac{D_x(t) - x_0}{l_x} \right) + \tan^{-1} \left( \frac{x_0}{l_x} \right) \right]. \tag{24} \]
This can be asymptotically approximated for $D_x \gg l_x$, which to leading order is

$$K_y(t) \sim V_0^2 T l_y \frac{l_x}{D_x(t)} \left[ \frac{\pi}{2} - \frac{l_x}{D_x(t) - x_0} + \tan^{-1} \left( \frac{x_0}{l_x} \right) \right]. \quad (25)$$

For large $t$, $D_x \sim t^{1/4}$, so that the long-time $K_y$ scales as

$$K_y(t) \sim \frac{1}{t^{1/4}} + O \left( \frac{1}{t^{1/2}} \right), \quad (26)$$

consistent with the drifter-derived scaling (dashed black line, Fig. 3b). Notice that the leading order scaling is $\sim 1/D_x(t)$ as long $V^2(x)$ decays offshore faster than $1/|x|$. This ensures that $\int_{-D_x}^{0} V^2(x) \, dx$ increases slowly enough for increasing $D_x$ to not effect the $K_y(t) \sim 1/D_x(t)$ scaling at leading order. Thus, unlike in the cross-shore where the offshore $U^2$ power-law determines the $K_x$ long-time dependence, the specific offshore shape of $V^2(x)$ does not determine the long-time alongshore diffusivity scaling $K_y \sim t^{-1/4}$, rather, cross-shore dispersion governs the long-time alongshore $K_y$ scaling.

5. Fickian Models of Surfzone Cross-shore Dispersion

As material dispersion is often modeled using a diffusion equation, the ability of a diffusion equation to simulate the drifter dispersion in inhomogeneous turbulence with a boundary is examined. With zero mean Eulerian flow, the one-dimensional diffusion equation for conserved tracer $\phi(t, x)$ is

$$\frac{\partial}{\partial t} \phi = \frac{\partial}{\partial x} \left[ \kappa_F(t, x) \frac{\partial}{\partial x} \phi \right], \quad (27)$$

where $\kappa_F(t, x)$ is the cross-shore and time-dependent Fickian diffusivity. Analogous to drifter dispersion, the domain is $x \leq 0$, with a shoreline ($x = 0$) no-flux boundary condition $d\phi/dx = 0$, analogous to drifter reflection. A delta-function initial condition $\phi(0, x) = \delta(x - X_0)$ corresponds
to drifters released from $X_0$. Analogous to drifter statistics, the tracer center of mass $\bar{X}_\phi(t)$ and eddy diffusivity $K_\phi$ are estimated from $\phi$ moments as

$$\bar{X}_\phi(t, X_0) = \phi_1(t, X_0)$$

$$K_\phi(t, X_0) = \frac{1}{2} \frac{d}{dt} \left\{ \phi_2(t, X_0) - [\phi_1(t, X_0)]^2 \right\},$$

where the $n$th moment is $\phi_n(t, X_0) = \int_{-\infty}^{0} x^n \phi(t, x, X_0) \, dx$. Here, the effect of cross-shore uniform and cross-shore varying Fickian diffusivity $\kappa_F$ on the tracer eddy diffusivity $K_\phi$ is examined to provide insight into the drifter derived eddy diffusivity $K_x(t, X_0)$.

### a. Cross-shore uniform, time-dependent Fickian diffusivity

In Spydell and Feddersen (2012b), surfzone drifter dispersion for drifters spanning a range of $X_0$ was simulated with a cross-shore uniform, but time-dependent Fickian diffusivity that allows for ballistic dispersion,

$$\kappa_F = \kappa(t) = \mathcal{U}^2 T_L [1 - \exp(-t/T_L)]$$

where $\mathcal{U}^2$ is the spatially-uniform cross-shore eddy velocity variance and $T_L$ is the spatially-uniform Lagrangian (decorrelation) time-scale in contrast to $T_{Lx}$ which parameterizes the maximum diffusivity. The diffusion equation (27) is solved for three $X_0 = -40, -80, 120$ m, $\mathcal{U}^2$ that matches that at $X_0 = -80$ m, and $T_L = 120$ s. The choice of $T_L = 120$ s is discussed in Section 6a and is approximately the surfzone averaged Eulerian time-scale. With this $\kappa_F$, the analytically-derived $\bar{X}_\phi(t, X_0)$ and $K_\phi(t, X_0)$ have release location $X_0$ dependent offshore drift $\bar{X}_\phi$, and a shoreline induced subdiffusive regime with asymptotic diffusivity reduced (by factor $(\pi - 2)/\pi < 1$) relative to no shoreline (Spydell and Feddersen 2012b). There are no inhomogeneous eddy velocity
Initially \((t < 100\) s), the center of mass position \(\bar{X}_{\phi}\) is constant for all three \(X_0\), consistent with model-drifter derived \(\bar{X}\) (Fig. 7a). By design, the short-time \(K_{\phi}\) matches well the drifter \(K_x(t, X_0)\) at \(X_0 = -80\) m (Fig. 7c), however the short-time \(K_{\phi}\) does not match \(K_x\) at other locations because \(U^2(x)\) varies. For larger times, drifter and Fickian-diffusion statistics have significant differences. The tracer \(\bar{X}_{\phi}\) monotonically drifts offshore eventually quite rapidly (colors in Fig. 7a) because the cross-shore constant \(U^2\) is relatively large. In contrast, at \(X_0 = -80\) m and \(X_0 = -120\) m, \(\bar{X}\) is onshore for \(t < 1000\) s before heading offshore at long-times much more slowly than the tracer center of mass \(\bar{X}_{\phi}\). The strong subdiffusive regime in the drifter-derived \(K_x\) is not present in the tracer \(K_{\phi}\), which asymptotes to a constant value larger than the long-time \(K_x\). Hence, a spatially uniform surfzone \(\kappa_F\) will over-estimate material flux onto the inner-shelf. These comparisons highlight the strong role that cross-shore inhomogeneous turbulence plays in drifter or tracer dispersion and thus cross-shore exchange.

**Fig. 7**

### b. A cross-shore and time dependent Fickian diffusivity

To represent the cross-shore inhomogeneous turbulence, a spatially-varying and time-dependent \(\kappa_F(x, t)\) is used with form

\[
\kappa_F(x, t) = U_{an}^2(x)T_L\left[1 - \exp\left(-t/T_L\right)\right],
\]

where \(U_{an}^2(x)\) (5) represents the cross-shore variable eddy velocity variance. The diffusion equation (27) is solved numerically with \(T_L = 120\) s and for \(X_0 = -40, -80, -120\) m. The cross-shore variable \(\kappa_F(x, t)\) results in much better agreement between \((\bar{X}(t, X_0), K_x(t, X_0))\) and \((\bar{X}_{\phi}, K_{\phi})\) (Fig. 7c,d) than that for cross-shore uniform \(\kappa_F\) (Fig. 7a,b). At short times \((t < 100\) s), the ballis-
tic dispersion is reproduced for all $X_0$, as $U_{\text{an}}^2(x)$ closely matches $U^2(x)$. At intermediate times ($10^2 < t < 10^3$ s), the $K_x$ maximum and time to maximum ($T_x$) are largely reproduced by $K_\phi$ (Fig. 7d). As $U_{\text{an}}^2(x)$ and $U^2(x)$ are similar, this implies that with (31), the effective Lagrangian time-scale $\bar{T}_{Lx} = 66$ s is reproduced. This reinforces the difference between $\bar{T}_{Lx}$ (which sets $K$) and the decorrelation-time $T_L$, and suggests that $\bar{T}_{Lx}$ is about half of $T_L$ ($\bar{T}_{Lx} \approx T_L/2$). At intermediate times ($10^2 < t < 10^3$ s), the $X_0$-dependent mean drift $\bar{X}$ is also reproduced by $\bar{X}_\phi$ (Fig. 7c). At long times ($t > 10^3$ s), the slow offshore drift velocity $\bar{U}$ and the long-time $K_x \sim t^{-1/2}$ is largely reproduced by $\bar{X}_\phi$ and $K_\phi$, in contrast to long-time rapid offshore drift and constant $K_\phi$ (Fig. 7a,b) for a spatially uniform $\kappa_F$. This indicates that the Fickian diffusivity (31) with cross-shore variable $U_{\text{an}}^2(x)$ and spatially-uniform $T_L$ within a diffusion equation (27) can reproduce the drifter-derived Lagrangian statistics in regions of inhomogeneous turbulence. It also indicates that this simple and computationally efficient model can be used to test scalings.

c. The effect of cross-shore decaying eddy velocity variance

Here, the relationship between the offshore decay of $U_{\text{an}}^2(x)$ and the long-time $K_\phi \sim t^\gamma$ power law is examined. The long-time drifter $K_x \sim t^{-1/2}$ scaling results from $U_{\text{an}}^2 \sim x^{-2}$ (Section 4c2). For a generalized $U_{\text{an}}^2 \sim x^{-\beta}$, similar reasoning leads to

$$K_x \sim t^{-\beta/(2+\beta)}.$$ (32)

This is analogous to scale-dependent relative diffusivity, where $\beta = -4/3$ yields $K_x \sim t^2$ or $D_x^2 \sim t^3$ for drifter separations (Richardson 1926).

The long-time $K_x$ scaling (32) is tested via numerical solution of (27) with Fickian diffusion.
of the form (31), but with analytic $U_{an}^2(x)$ with $\beta$ dependence ie,

$$U_{an}^2(x) = \frac{U_0^2}{1 + |(x - x_{max})/l_x|^{\beta}}, \quad (33)$$

such that far-offshore $U_{an}^2 \sim |x|^{-\beta}$ with larger $\beta$ corresponding eddy velocities decaying more rapidly offshore. Numerical solutions use the same $T_L = 120$ s, $X_0 = -40$ m, $U_0^2 = 0.015$ m$^2$s$^{-2}$, and $\beta$ varies from 0.5 to 3. For each $\beta$, $K_\phi(t)$ is estimated (29). Assuming a long-time power-law for $K_\phi \sim t^\gamma$, the exponent $\gamma$ is estimated as $\gamma = d\{\log[K_\phi(t)]\}/d\{\log[t]\}$ averaged over $9000 \leq t \leq 1 \times 10^4$ s with derivatives estimated by finite difference.

The more rapidly $U_{an}^2(x)$ decays offshore, the more negative the exponent $\gamma$ (red dots in Fig. 8) and the more strongly subdiffusive the long-time $K_x$. The exponent $\gamma$ is largely consistent with the long-time $K_x$ scaling (32) (compare red dots to the dashed curve). Differences between $\gamma$ estimated using $\phi$ and the scaling (32) are due to differences between $U_{an}^2(x)$ (33) used in the diffusion equation, and the form ($|x|^{-\beta}$) that leads to (32). Additionally, the diffusion equation includes shoreline effects not present in (32). Overall, however, the similarity between $\gamma$ and the scaling (32) indicates that (32) can be used to estimate the long-time diffusivity in other regions with spatially inhomogeneous turbulence.

6. Discussion

For point drifter releases in a inhomogeneous turbulent surfzone, the short-time and long-time mean drift $\bar{X}$ and the short-time, maximum, and long-time cross- and alongshore diffusivities $K$ can be scaled. These scalings require knowledge of the cross-shore structure of the cross-shore $U^2(x)$ and alongshore $V^2(x)$ eddy velocity variances, and the constant effective Lagrangian time-scales $(\bar{T}_{Lx}, \bar{T}_{Ly})$. However, only one case of normally-incident wave conditions with zero
alongshore current was examined here. The presence of a sheared alongshore current will increase
$K_y$ substantially via shear dispersion (Spydell and Feddersen 2012b), but potentially decrease $K_x$
via mixing suppression induced by the eddy propagation speed and the mean current difference
(Spydell 2016). The effect of other parameters, such as variable beach profiles, incident wave
height $H_s$, period, or directional spread $\sigma_\theta$, on $\bar{X}$ and $K_i$ are not yet understood.

\hspace{1cm} a. Scaling the “bulk” Lagrangian Timescale

In scaling $K_x$ and $K_y$, cross-shore uniform effective Lagrangian time-scales $\bar{T}_{Lx}$ and $\bar{T}_{Ly}$ were
appropriate, but were found by a fit. However, the effective Lagrangian time-scales are generally
unknown for other surfzones or other applications. For example $\bar{T}_{Lx}$ may scale with incident $H_s$
or beach slope. For the simulation presented here, the Eulerian time-scale $T_E$ is used to derive the
effective Lagrangian time-scale following concepts developed by Middleton (1985).

In homogeneous turbulence, dispersion can be in either the “frozen field” or “fixed float” limits
(e.g., Middleton 1985; Lumpkin et al. 2002). In the “frozen field” limit, drifters sample multiple
eddies before the eddy changes significantly, so that the ratio of Lagrangian to Eulerian timescales
is small ($T_L/T_E \ll 1$), and $K^\infty \propto U_L E$ where $L_E$ is the Eulerian eddy size (Middleton 1985). In
the “fixed float” limit, temporal Lagrangian fluctuations are caused by Eulerian velocity fluctua-
tions and $T_L/T_E \approx 1$ resulting in $K^\infty = U^2 T_E$. These limits are distinguished using the Eulerian
“Middleton” parameter $\alpha$ (Middleton 1985) defined as $\alpha = U T_E / L_E$, where small $\alpha < 0.3$ indicates
“fixed float” and larger $\alpha > 0.3$ represents “frozen field” limits (Lumpkin et al. 2002). Surfzone
drifter observations spanning different beaches and different wave conditions suggest that there is
not a single value of surfzone $\alpha_{x,y}$ and it can potentially vary between “fixed float” and “frozen-
field” regimes (Spydell et al. 2014). To assess whether for this simulation turbulent dispersion is
in the fixed float or frozen field regimes, the cross-shore dependent Middleton parameters

\[ \alpha_x(x) = U(x) \frac{T_{Ex}(x)}{L_{Ex}(x)}, \quad \alpha_y(x) = V(x) \frac{T_{Ex}(x)}{L_{Ex}(x)} \]  

(34)

are calculated where the Eulerian time-scales \( T_{Eu_i}(x) \) (A1b) and length-scales \( L_{Eu_i}(x) \) (A1a) are
estimated as described in Appendix A. The Eulerian length-scales are mostly cross-shore uniform
and are the order of the surfzone width (not shown). At all cross-shore locations both \( \alpha_x \) and \( \alpha_y \)
are small (\( \leq 0.04 \), Fig. 9a,b), much less than the 0.3 value separating “fixed float” and “frozen
field” regimes. Although the Middleton parameter was developed for isotropic and homogeneous
turbulence, this suggests that in the region spanning the inhomogeneous surfzone to inner-shelf is
in the “fixed float” regime, where the Lagrangian and Eulerian time-scales are similar.

The cross-shore Eulerian velocity time-scale \( T_{Ex} \) varies between approximately 120–200 s for
regions not near the shoreline (\( x < -25 \) m, Fig. 9c). Although 2-3 times larger than the effective
Lagrangian time-scale \( \bar{T}_{Lx} \) (Fig. 9c), \( T_{Ex} \) is similar to and slightly larger than the cross-shore
constant Lagrangian time-scale \( T_{Lx} = 120 \) s (a true decorrelation time) used in a diffusion equation
that well reproduced the Lagrangian statistics (Section 5). Thus, consistent with the small \( \alpha_x \)
found in the surfzone, the Lagrangian time-scale will be approximately equal to the cross-shore
averaged (represented by a bar) Eulerian time-scales, \( i.e., \bar{T}_{Lx} \approx \bar{T}_{Ex} \). It also implies that the
effective Lagrangian time-scale, which sets \( \mathcal{K}_x \), can be written as \( \bar{T}_{Lx} \approx 0.5 T_{Lx} \approx 0.5 \bar{T}_{Ex} \). Recall
\( \bar{T}_{Lx} \) is not a decorrelation time. In the alongshore, the relationship between \( \bar{T}_{Ly}, T_{Ly} \), and \( T_{Ey} \)
is not clear because alongshore diffusion simulations with an associated \( T_{Ly} \) were not performed
as alongshore dispersion cannot be represented as 1D diffusion due to coupling with cross-shore
dispersion. However, within the surfzone, the alongshore Eulerian and effective Lagrangian time-
scale are similar \( T_{Ey} \approx \bar{T}_{Ly} \) (Fig. 9d). The small surfzone \( \alpha_y \) (Fig. 9b) indicates fixed float conditions where \( T_{Ly} \approx T_{Ey} \). This suggests that the time-scale that sets the alongshore diffusivity maximum is closer to the decorrelation Lagrangian time-scale, i.e., \( \bar{T}_{Ly} \approx T_{Ly} \), potentially because alongshore dispersion is unbounded. Thus, surfzone dispersion on alongshore uniform coasts with no mean flows can be well simulated with knowledge of the cross-shore structure of the eddy velocity variance \( \langle U^2(x) \rangle \) and \( \langle V^2(x) \rangle \) and Eulerian time-scales.

\[ b. \text{Relationship to previous surfzone drifter studies} \]

For short to intermediate times \( t \leq 1000 \) s, the \( K_x \) estimated here are relatively consistent with \( K_x \) estimated from in situ (Spydell et al. 2007; Brown et al. 2009; Spydell et al. 2009a) and numerical (Spydell and Feddersen 2009) surfzone drifter dispersion. Many of these studies had non-zero mean alongshore current and so the focus here is on \( K_x \). In both in situ and numerical, \( K_x \) is initially ballistic, reaches a maximum, then decreases (i.e., is subdiffusive). In some cases, the diffusivity estimator had a bias towards constant long-time diffusivity (Spydell et al. 2007), that reduced subdiffusive \( K_x \) behavior. The shoreline was argued capable of inducing the observed subdiffusive \( K_x \) for \( t < 10^3 \) s (Spydell and Feddersen 2012b). However, in these works, drifters were released over many \( X_0 \), and the derived \( K_x \) are an average over \( X_0 \), blurring out inhomogeneous turbulence effects. The relatively short \( t < 10^3 \) s observed and modeled drifter trajectories resulted in unclear long-time diffusivity behavior, as discussed by Spydell and Feddersen (2012b). Here, long-time subdiffusive behavior is clear in the long \( (2 \times 10^4 \) s) trajectories. Much longer in situ trajectories and many more drifters are required to constrain in situ long-time diffusivity estimates. However, at these long times, wind-driven, tidal, internal wave, or other processes such as
the interaction of stratification with transient rip currents (Kumar and Feddersen 2017) will induce
dispersion and cross-shelf exchange.

c. Applicability to other oceanographic regions

The principal result here is that strong gradients in the eddy velocity variance have significant
effects on the dispersion and mean drift of drifters. Although the inhomogeneous turbulence of
the surfzone to inner-shelf is considered here, these results may be more broadly applicable, as
many regions of the ocean have gradients in eddy velocities that are order one relative to the eddy
size. For example, a horizontal diffusivity maximum and subsequent decrease (subdiffusion) in the
Antarctic Circumpolar Current was attributed (Klocker et al. 2012) to the mixing supression in-
duced by the mean current and eddy propagation velocity difference (e.g., Ferrari and Nikurashin
2010). Similarly, using Global Drifter Program data for the surface and mixed-layer, diffusivity
maxima $K$ occurred between 3-10 days with a subsequent decrease (Zhurbas et al. 2014). Defining
$K_\infty$ as the average diffusivity over 15-20 days, Zhurbas et al. (2014) showed that $K/K_\infty$ maps
strongly resemble eddy kinetic energy maps, with both having the largest values in the ACC and
Western Boundary Current extensions. Zhurbas et al. (2014) also attributed the $K/K_\infty$ ratio varia-
tion to the mixing suppression mechanism invoked by (Klocker et al. 2012). However, in both of
these cases, the gradients in eddy kinetic energy were strong and thus inhomogeneous turbulence
may have contributed to the observed subdiffusion. These regions did not have boundaries. In the
Santa Barbara Channel, a region with boundaries and variations in eddy kinetic energy, diffusivity
maxima and subsequent decrease also was noted (Dever et al. 1998). Non-zero and non-uniform
mean currents can alter the dispersion: shear dispersion would enhance the along-flow dispersion
and mixing suppression by unequal eddy propagation and mean current speeds would decrease the diffusivity. Other mechanisms influencing the diffusivity include slow time modulations of the mean flow (Qian et al. 2014). Here, however, the mean currents were zero and these mechanisms were not present. In environments with both sheared mean currents and gradients in eddy velocities, shear dispersion, mixing suppression, and inhomogeneous turbulence all could affect the diffusivity, but the relative contribution of each mechanism is not understood.

7. Summary

Drifters were tracked in an idealized surfzone simulation with random normally-incident but directionally spread waves using the wave-resolving FunwaveC model. This simulation had no mean currents but a strong cross-shore inhomogeneous eddy field. Lagrangian statistics were calculated with many more drifters and over much longer ($\approx 10^4$ s) trajectories than in previous observational or modeling studies, allowing for the cross-shore release location and time-dependence to be accurately studied.

The cross-shore inhomogeneity of the eddy field affects both the mean drift $\bar{X}$, and the cross- ($K_x$) and alongshore ($K_y$) diffusivity. The short-time cross- and alongshore diffusivity is locally ballistic and the mean cross-shore drift is toward the cross-shore eddy velocity variance maximum, consistent with theory. At intermediate times, $K_x$ and $K_y$ reach a maximum and subsequently decrease. Hence, both cross- and alongshore dispersion are subdiffusive. The maximum $K_x$ and $K_y$ are parameterized in terms of a release-location dependent, cross-shore averaged eddy velocity variance, taking into account cross-shore inhomogeneity, and a cross-shore uniform effective Lagrangian time-scale. The time to the diffusivity maxima is related to the release location eddy
velocity and the distance from release location across the maximum of the eddy velocity variance. At long-times, the subdiffusive diffusities scale as $K_x \sim t^{-1/2}$ and $K_y \sim t^{-1/4}$. This time-decay is related to the cross-shore decay of the eddy velocity variance. The different long-time $K_x$ and $K_y$ time-dependence is because alongshore dispersion is coupled to cross-shore dispersion.

Cross-shore drifter transport and dispersion was consistent with a diffusion equation that included a time- and space-dependent Fickian diffusivity, that locally has ballistic to Brownian diffusion time-dependence. The Fickian diffusivity is related to the cross-shore eddy velocity variance and with this spatial dependence, the short, intermediate, and long time behavior of $\bar{X}$ and $K_x$ are reproduced. The Eulerian Middleton parameter $\alpha$ is small in this region indicating fixed float dispersion and that the effective Lagrangian time-scale is related to the (surfzone averaged) Eulerian time-scale. In these simulations there was no mean flow and so the effects of shear dispersion and mixing suppression was not present. The effects of inhomogeneous turbulence on drifter dispersion spanning the surfzone to inner-shelf are likely also relevant for other oceanographic regions such as the ACC and Western Boundary Current extensions.

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APPENDIX A Eulerian Time-scales and Length-scales

From model output, cross- and alongshore Eulerian velocity alongshore cyclic wavenumber
(k_y) and frequency (f) spectra, S_u(k_y, x) and S_v(f, x) respectively, are calculated at all cross-shore x locations from which Eulerian length-scales (L_E) and time-scales (T_E) are estimated. The frequency spectra S_u(f, x) and S_v(f, x) are red with f^{-2} dependence for 0.002 < f < 0.04 Hz at all x (not shown). Thus, over these 25–500 s time-scales, the Eulerian eddy field is roughly consistent with a simple AR1 stochastic process forced with white noise (i.e., the breaking wave eddy forcing, Clark et al. 2012), consistent with stochastic surfzone drifters simulations (Spydell and Feddersen 2012b). The magnitude of S_u(k_y, x) and S_v(k_y, x) depends strongly on cross-shore location (Fig. A1) with maxima at x_{U_{mx}} and x_{V_{mx}}, respectively, consistent with U(x) and V(x) (Fig. 1c). The S_u(k_y, x) and S_v(k_y, x) have similar spectral shapes (Fig. A1a,b) for k_y > 0.01 cpm (cycles per meter). However at small wavenumber (k_y < 0.005 cpm), particularly closer to the shoreline, S_v(k_y, x) is larger and flatter than S_u(k_y) indicating anisotropy. The alongshore wavenumber spectra S_u(k_y, x) and S_v(k_y, x) at suggest the presence of both inverse-energy cascade (\sim k_y^{-5/3}) and enstrophy cascade (\sim k_y^{-3}) regions (Fig. A1, solid and dashed lines) expected for 2D turbulence forced at a single wavenumber (e.g., Salmon 1998).

The cross-shore dependent Eulerian eddy time- T_E(x) and length-scales L_E(x) are estimated from the alongshore wavenumber and frequency spectra via

\[ L_{Eu_i}(x) = 5 \mathcal{U}_i^2 \left[ \int_0^\infty k_y S_{u_i}(k_y, x) \, dk_y \right]^{-1} \]  
\[ T_{Eu_i}(x) = \frac{1}{4} \mathcal{U}_i^{-2} S_{u_i}(f = 0, x). \]  

The definition of T_E is the spectral version of the Eulerian velocity decorrelation time-scale used in Middleton (1985). The length-scale L_E (A1a) differs from Middleton (1985), who used L_E = 2\mathcal{U}_i^{-2} \int k^{-1} S(k) \, dk, because at small k, S(k) \neq 0 (Fig. A1) leading to large L_E. The value of 5 is included in (A1a) so that the L_E from (A1a) and that of Middleton (1985) are approximately equal.
for the same wave-number spectra $S_u(k_y)$. Thus, the $\alpha$ calculated here can be directly compared to previously published (Middleton 1985; Lumpkin et al. 2002) values.
REFERENCES


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Figure Captions

FIG. 1. (a) Depth $h$, (b) significant wave height $H_s$, (c) cross-shore eddy velocity variance $U^2$ (black curve), and (d) alongshore eddy velocity variance $V^2$ (black curve) versus cross-shore distance $x$. In (c,d), analytic forms for the eddy velocity variance ($U^2_{an}$ and $V^2_{an}$) are shown as dashed red lines. The shoreline is at $x = 0$, and the cross-shore location of the maximum cross- $x_{U_{max}}$ and alongshore eddy velocity $x_{V_{max}}$ is indicated (gray vertical lines). The gray dash-dotted vertical line denotes the offshore limit of the surfzone $x_{sz}$ – defined as the $x$ location of maximum $H_s$.

FIG. 2. Virtual drifter positions (colored dots) for one of the 53 realizations at various times (indicated above each panel). Drifters are colored by the initial cross-shore position $X_0$: offshore of the surfzone (blue), outer edge of the surfzone (green), and near the shoreline (red). The gray dash-dotted vertical line indicates the surfzone boundary $x_{sz}$. The background color is the instantaneous vorticity.

FIG. 3. (a) The mean cross-shore trajectory $\bar{X}$ (7), (b) mean cross-shore drift velocity $\bar{U}$ (8), the (c) cross- ($U^2_L$) and (d) alongshore $V^2_L$ Lagrangian velocity variance (10), and the (e) cross- ($K_x$) and (f) alongshore ($K_y$) eddy diffusivities (11) versus time $t$. Colors indicate initial cross-shore drifter location $X_0$: from the shoreline (red) to offshore (blue). In (a) the location of the maximum Eulerian cross-shore eddy velocity $x_{U_{max}}$ is indicated by the gray line. In (a) and (b), the dashed black is a $|\bar{X}(t)| \sim t^{1/4}$, and $|\bar{U}| \sim t^{-3/4}$, respectively, prediction. In (b), open circles indicate the time of strongest offshore velocity, i.e., the time the shoreline is felt. Various scaling laws are indicated in (c), (d), (e) and (f).

FIG. 4. (a) Cross- ($K_x$) and (b) alongshore ($K_y$) eddy diffusivity, scaled by the release location $X_0$ cross-$U^2$ and alongshore eddy velocity variance $V^2$, respectively, versus time $t$. Colors indicate the initial release location and are the same as in Fig. 3. The dashed black line, $= t$, indicates ballistic dispersion.

FIG. 5. The cross-shore drift acceleration $d^2\bar{X}/dt^2$ versus the cross-shore gradient of the cross-shore squared eddy velocity $dU^2(x)/dx$ at $t = 10$ s. Colors indicate initial cross-shore drifter location $X_0$: from the shoreline (red) to offshore (blue). The 1-to-1 line is dashed.

FIG. 6. The maximum (a) cross-shore diffusivity $K_x$ and (b) maximum alongshore diffusivity $K_y$ as a function of cross-shore release location $X_0$ (black curves). The red curves in (a,b) are the
parameterized \( \bar{U}^2\bar{T}_{Lx} \) (17) and \( \bar{V}^2\bar{T}_{Ly} \) (20). The time to maximum (c) cross-shore diffusivity \( T_x \) and (d) alongshore diffusivity \( T_y \) versus cross-shore release location \( X_0 \) (black curves). The red curves in (c,d) are parameterizations given by (19). The surfzone extent \( (x_{sz}) \), and the location of maximum cross- \( (x_{U_{mx}}) \), and alongshore \( (x_{V_{mx}}) \), eddy velocity variances are indicated by vertical dashed, dash-dotted, and dotted lines, respectively.

**Fig. 7.** (a) The cross-shore mean position \( \bar{X}_\phi \) and (b) the cross-shore diffusivity \( K_\phi \) versus time \( t \) for the spatially constant Fickian diffusion model for 3 release locations \( X_0 \) (colors, see legend). Black curves are surfzone turbulence derived quantities. (c,d) Same as in (a,c) but for the cross-shore dependent Fickian diffusion model.

**Fig. 8.** The power-law exponent \( \gamma \) of the long-time cross-shore diffusivity \( K_\phi \sim t^\gamma \) versus the exponent of offshore eddy variance decay \( \bar{U}^2 \sim x^{-\beta} \) for both analytic prediction ((32), dashed curve) and numerical solution (red dots).

**Fig. 9.** (a) The Middleton parameter \( \alpha_x \) (34) versus \( x \). (c) The Eulerian cross-shore velocity time-scale \( T_{Ex} \) (A1b) (blue) and cross-shore uniform effective Lagrangian time-scale \( T_{Lx} \) (black line) versus \( x \). (b,d) Alongshore versions of panels (a,c), i.e., (b) \( \alpha_y \) and (d) \( T_{Ey} \) and \( T_{Ly} \). The surfzone extent \( x_{sz} \) (vertical dashed), and the maximum cross- and alongshore eddy velocities \( (x_{U_{mx}}, x_{V_{mx}}, \text{dotted}) \) are indicated.

**Fig. A1.** The spectrum of (a) cross- \( S_u(k_y, x) \) and (b) alongshore velocities \( S_u(k_y, x) \) versus cyclic alongshore wavenumber \( k_y \). Spectra are taken at various cross-shore locations (colors): from the shoreline (red) to offshore (blue) where the edge of the surfzone is \( x \approx -90 \) m. Inverse-energy cascade \( (k_y^{-5/3}) \) and enstrophy cascade \( (k_y^{-3}) \) scalings are shown as solid and dashed lines, respectively.
FIG. 1. (a) Depth $h$, (b) significant wave height $H_s$, (c) cross-shore eddy velocity variance $u^2$ (black curve), and (d) alongshore eddy velocity variance $v^2$ (black curve) versus cross-shore distance $x$. In (c,d), analytic forms for the eddy velocity variance ($u^2_{an}$ and $v^2_{an}$) are shown as dashed red lines. The shoreline is at $x = 0$, and the cross-shore location of the maximum cross- $x_{U_{mx}}$ and alongshore eddy velocity $x_{V_{mx}}$ is indicated (gray vertical lines). The gray dash-dotted vertical line denotes the offshore limit of the surfzone $x_{sz}$—defined as the $x$ location of maximum $H_s$. 
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