Hydrodynamics of Spur and Groove Formations on a Coral Reef

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Abstract

Spur-and-groove formations are found on the fore reefs of many coral reefs worldwide. Although these formations are primarily present in wave-dominated environments, their effect on wave-driven hydrodynamics is not well understood. A two-dimensional, depth-averaged, phase-resolving non-linear Boussinesq model (funwaveC) was used to model hydrodynamics on a simplified spur-and-groove system. The modeling results show that the spur-and-groove formations together with shoaling waves induce a nearshore Lagrangian circulation pattern of counter-rotating circulation cells. The mechanism driving the modeled flow is an alongshore imbalance between the pressure gradient and nonlinear wave terms in the momentum balance. Variations in model parameters suggest the strongest factors affecting circulation include spur-normal waves, increased wave height, weak alongshore currents, increased spur height, and decreased bottom drag. The modeled circulation is consistent with a simple scaling analysis based upon the dynamical balance of the nonlinear wave term, pressure gradient and bottom stress. Model results indicate that the spur-and-groove formations efficiently drive circulation cells when the alongshore spur-and-groove wavelength allows for the effects of diffraction to create alongshore differences in wave height without changing the mean wave angle.
1. Introduction

Coral reefs provide a wide and varied habitat that supports some of the most diverse assemblages of living organisms found anywhere on earth [Darwin, 1842]. Reefs are areas of high productivity because they are efficient at trapping nutrients, zooplankton, and possibly phytoplankton from surrounding waters [Odum and Odum, 1955; Yahel et al., 1998]. The hydrodynamics of coral reefs involve a wide range of scales of fluid motions, but for reef scales of order 100 m to 1000 m, surface wave-driven flows often dominate [e.g., Monismith, 2007].

Hydrodynamic processes can influence coral growth in several ways [Chappell, 1980]. Firstly, waves and mean flows can suspend and transport sediments. This is important because suspended sediment is generally recognized as an important factor that can negatively affect coral health [Buddemeier and Hopley, 1988; Acevedo et al., 1989; Rogers, 1990; Fortes 2000; Fabricius, 2005]. Often, suspended sediment concentrations are highest along the reef flat, and are much lower in offshore ocean water [Ogston et al., 2004; Storlazzi et al., 2004; Storlazzi and Jaffe, 2008]. Secondly, forces imposed by waves can subject corals to high drag forces breaking them, resulting in trimming or reconfiguration of the reef [Masselink and Hughes, 2003; Storlazzi et al., 2005]. Thirdly, the rates of nutrient uptake on coral reefs [Atkinson and Bilger, 1992; Thomas and Atkinson, 1997], photosynthetic production and nitrogen fixation by both coral and algae [Dennison and Barnes, 1988; Carpenter et al., 1991], and particulate capture by coral [Genin et al., 2009] increase with increasing water motion.

One of the most prominent features of fore reefs are elevated periodic shore-normal ridges of coral (“spurs”) separated by shore-normal patches of sediment (“grooves”), generally located offshore of the surf zone [Storlazzi et al., 2003]. These features, termed “spur-and-
“groove” (SAG) formations, have been observed in the Pacific Ocean [Munk and Sargent, 1954; Cloud, 1959; Kan et al., 1997, Storlazzi et al., 2003; Field et al., 2007], the Atlantic Ocean [Shinn et al., 1977, 1981], the Indian Ocean [Weydert, 1979], the Caribbean Sea [Goreau, 1959; Roberts, 1974; Geister, 1977; Roberts et al., 1980; Blanchon and Jones, 1995, 1997], the Red Sea [Sneh and Friedman, 1980], and other locations worldwide. SAG formations are present on fringing reefs, barrier reefs, and atolls. Typical SAG formations off the fringing reef of southern Moloka‘i, Hawai‘i, are shown in Figure 1 and Figure 2.

The alongshore shape of the SAG formations varies from smoothly varying rounded spurs [Storlazzi et al., 2003], to nearly flat spurs with shallow rectangular channel grooves [Shinn et al., 1963, Cloud, 1959], or deeply cut rectangular or overhanging channels often called buttresses [Goreau, 1959]. The scales of SAG formations vary worldwide, but in general spur height \( h_{spr} \) is of order 0.5 m to 10 m, SAG alongshore wavelength \( \lambda_{SAG} \) is of order 5 m to 150 m, the width of the groove \( W_{grv} \) is of order 1 m to 100 m, and SAG formations are found in depths \( h \) from 0 m to 30 m below mean sea level, [Munk and Sargent, 1954; Roberts, 1974; Blanchon and Jones, 1997; Storlazzi et al., 2003].

Although the geometric properties of SAG formations are well documented, analysis of their hydrodynamic function has been limited. On Grand Cayman [Roberts, 1974] and Bikini Atoll [Munk and Sargent, 1954], SAG formations were shown to be related to incoming wave energy; high incident wave energy areas have well-developed SAG formations, whereas those with low incident wave energy have little to no SAG formations. The spur and groove formations of southern Moloka‘i, Hawai‘i, have been well-characterized; and incident surface waves appear to exert a primary control on the SAG morphology of the reef. [Storlazzi et al., 2003; Storlazzi et
Spurs are oriented orthogonal to the direction of predominant incoming refracted wave crests, and $\lambda_{\text{SAG}}$ is related to wave energy [Munk and Sargent, 1954; Emry et al., 1949; Weydert, 1979; Sneh and Friedman, 1980; Blanchon and Jones, 1995]. SAG formations are proposed to induce a cellular circulation serving to transport debris away from the reef along the groove [Munk and Sargent, 1954]; however, no field or modeling studies have been carried out to assess this circulation. Although the relationship between SAG orientation and incoming wave orientation, and the relationship between $h_{\text{spr}}, \lambda_{\text{SAG}}$, and incoming wave energy are qualitatively known, the mechanism for these relationships has not been investigated.

The primary purpose of the present work is to examine the hydrodynamics of a typical fore reef system (seaward of the surf zone) with SAG formations to determine the effects of the SAG formations on the shoaling waves and circulation. To address this question, a phase resolving nonlinear Boussinesq model (Section 2) was used with idealized SAG bathymetry and site conditions from Moloka‘i, Hawai‘i (Section 3). The model shows that SAG formations induce Lagrangian circulation cells (Section 4.1). A mechanism for this circulation in terms of the momentum balance (Section 4.2), the role of various hydrodynamic and geometric parameters (Section 4.3), and the effect of spatially variable drag coefficient (Section 4.4), are investigated. A discussion follows on the relative effect of an open back reef on the SAG-induced circulation (Section 5.1), the hydrodynamics of different SAG wavelengths (Section 5.2), and the SAG induced-circulation and potential three-dimensional effects (Section 5.3), with conclusions in Section 6.
2. The Boussinesq Wave and Current Model

A time-dependent Boussinesq wave model, funwaveC, which resolves individual waves and parameterizes wave breaking, is used to numerically simulate velocities and sea surface height on the reef, [Feddersen, 2007; Spydell and Feddersen, 2009; and Feddersen et al. 2011]. The model Boussinesq equations [Nwogu, 1993] are similar to the nonlinear shallow water equations but include higher order dispersive terms. The equation for mass (or volume) conservation is:

$$ \frac{\partial \eta}{\partial t} + \nabla \cdot [(h + \eta)u] + \nabla \cdot M_d = 0, $$

where \( \eta \) is the instantaneous free surface elevation, \( t \) is time, \( h \) is the still water depth, \( M_d \) is the dispersive term, and \( u(u,v) \) is the instantaneous horizontal velocity at the reference depth \( z_r = -0.531h \) (approximately equal to the depth averaged velocity for small \( kh \)), where \( z = 0 \) at the still water surface. The momentum equation is

$$ \frac{\partial u}{\partial t} + u \cdot \nabla u = -g \nabla \eta + F_d + F_{br} - \frac{\tau_b}{\rho(h + \eta)} - \nu_{bi} \nabla^4 u - F_s, $$

where \( g \) is the gravitational constant, \( F_d \) are the higher-order dispersive terms, \( F_{br} \) are the breaking terms, \( \tau_b \) is the instantaneous bottom stress, and \( \nu_{bi} \) is the hyperviscosity for the biharmonic friction (\( \nabla^4 u \)) term, and \( F_s \) is the surface forcing. The dispersive terms \( M_d \) and \( F_d \) are given by equations 25a and 25b in Nwogu [1993]. The bottom stress is parameterized with a quadratic drag law

$$ \tau_b = \rho C_d u |u|, $$

with the nondimensional drag coefficient \( C_d \) and density \( \rho \). The effect of wave breaking on the momentum equations is parameterized as a Newtonian damping where
\[ F_{br} = \frac{1}{(h + \eta)} \nabla \cdot [v_{br}(h + \eta)\mathbf{v}], \tag{4} \]

where \(v_{br}\) is the eddy viscosity associated with the breaking waves [Kennedy et al., 2000]. When \(\partial \eta / \partial t\) is sufficiently large (i.e., the front face of a steep breaking wave), \(v_{br}\) becomes non-zero.

Additional details of the funwaveC model are described by [Feddersen, 2007; Spydell and Feddersen, 2009; and Feddersen et al., 2011].

Post processing of the instantaneous model velocity and sea-surface elevation output were conducted to separate the Eulerian, Lagrangian and Stokes drift velocities [e.g., Longuet Higgins 1969; Andrews & McIntyre, 1978]:

\[
U_E = \bar{\mathbf{u}}, \tag{5}
\]

\[
U_L = \frac{(h + \eta)}{\bar{h} + \eta} \mathbf{u}, \tag{6}
\]

\[
U_S = U_L - U_E, \tag{7}
\]

where, an over bar (\(\bar{\cdot}\)) indicates phase (time) averaging, \(U_E(U_E, V_E)\) is the mean Eulerian velocity, \(U_L(U_L, V_L)\) is the mean Lagrangian velocity, and \(U_S(U_S, V_S)\) is the Stokes drift. This form for \(U_S\) is compared to the linear wave theory form in Appendix A. The wave height \(H\) can be approximated from the variance of the surface [e.g., Svendsen, 2007]:

\[
H = \sqrt{8\langle \eta'^2 \rangle}, \tag{8}
\]

where \(\eta = \bar{\eta} + \eta'\). The mean wave direction \(\theta\) is given by,

\[
\tan 2\theta = \frac{2C_{uv}}{C_{uu} - C_{vv}}, \tag{9}
\]
where the variance \(\text{Var}(C_{uu}, C_{vv})\) and covariance \(C_{uv}\), are used with a monochromatic wave field, and are equivalent to the spectral definition given by *Herbers et al.*, [1999], and \(\theta = 0\) corresponds to normally incident waves. Although realistic ocean waves are random, monochromatic waves are used here for simplicity and to highlight the linkage of the wave shoaling on SAG bathymetry with the resulting circulation. A cross-shore Lagrangian circulation velocity \(U_c\) is defined as:

\[
U_c = U_L \cos(\varphi),
\]

where \(\varphi\) is the angle between the x and y components of \(U_L\). In the presence of a strong alongshore current, cross-shore circulation is negligible \((\varphi \approx \pi/2)\) and \(U_c\) will approach zero; while in the presence of strong cross-shore current \((\varphi \approx 0)\), \(U_c\) will approach \(U_L\).

Under steady-state mean current conditions, the phase averaged unsteady \(\partial \mathbf{u}/\partial t\) and dispersive \((F_d)\) terms in the Boussinesq momentum equations (Eq. 2) are effectively zero. Additionally, the velocity \(\mathbf{u}\) can be decomposed into mean \((\mathbf{u})\) and wave \((\mathbf{u}')\) components, essentially a Reynolds decomposition

\[
\mathbf{u} = \mathbf{u} + \mathbf{u}',
\]

and the phase-averaged nonlinear term of Eq. 2 becomes (with the use of Eq. 5):

\[
\mathbf{u} \cdot \nabla \mathbf{u} = (\mathbf{u} + \mathbf{u}') \cdot \nabla (\mathbf{u} + \mathbf{u}') = U_E \cdot \nabla U_E + \mathbf{u}' \cdot \nabla \mathbf{u}'.
\]

The phase averaged momentum equation can then be written as:

\[
U_E \cdot \nabla U_E + \mathbf{u}' \cdot \nabla \mathbf{u}' = -g \nabla \tilde{\eta} + \mathbf{F}_{br} - \frac{\tau_b}{\rho(h + \eta)} - U_{pi} \nabla^4 U_E - \mathbf{F}_s.
\]
The effect of the waves on the mean Eulerian velocity is given by the nonlinear wave term \((\mathbf{u}^\prime \cdot \nabla \mathbf{u}^\prime)\). This is analogous to a radiation stress gradient on the mean Lagrangian velocity, but without the effect of the free surface. The phase averaged bottom stress follows from Eq. 3:

\[
\bar{\tau}_b = \rho C_d |\mathbf{u}| \tag{14}
\]

In a weak current regime, where \( U_E / \sigma_u \) is small, where \( \sigma_u^2 \) is the total velocity variance, and away from the surf zone where \( \eta \ll h \), the bottom stress is proportional to the mean velocity,

\[ \bar{\tau}_b \propto U_E. \]  \citep{Feddersen2000}.

### 3. Model Setup and Conditions

#### 3.1 Model SAG Bathymetry

An idealized and configurable SAG bathymetry for use in numerical experiments was developed based on well-studied SAG formations on the southwestern coast of Moloka`i, Hawai`i (approximately 21°N, 157°W). High-resolution Scanning Hydrographic Operational Airborne Lidar Survey (SHOALS) laser-determined bathymetry data were utilized in combination with previous studies in the area \citep{Field2007}. The reef flat, with an approximate 0.3% slope and water depths ranging from 0.3 to 2.0 m, extends seaward from the shoreline to the reef crest (Figure 2, \( x < 400 \text{m} \)) \citep{Storlazzi2011}. Shore-normal ridge-and-runnel structure characterizes the outer reef flat. Offshore of the reef crest, from depths of 3 to 30 m lies the fore reef that is generally characterized by an approximately 7% average slope \((\beta_f)\) and shore-normal SAG structures covered by highly variable percentages of live coral (Figure 2) \citep{Storlazzi2011}. Note the SAG formations have a roughly coherent \( \lambda_{SAG} \) and cross-shore position, yet with natural variability.
Analysis of the SHOALS bathymetric data used in Storlazzi et al. [2003] was conducted, of the fringing reef of southern Moloka‘i from Kaunakakai west approximately 18.5 km to the western extent of the island. Alongshore bathymetric profiles taken at the 5, 10, 15, and 20 m depth isobaths were analyzed using a zero crossing method (similar to wave height routines). Of a total 784 measured SAG formations across all depths, the results show a mean $\lambda_{SAG}$ of 91 m, and a mean $h_{spr}$ of 3.0 m, (Figure 3). SAG formations generally had larger $\lambda_{SAG}$ and $h_{spr}$ at deeper depths, a conclusion also noted in Storlazzi et al. [2003].

A selection of 10 prominent SAG formations from this same area of southern Moloka‘i, from areas with documented active coral growth in Field et al. [2007] was used to further characterize $\lambda_{SAG}, h, W_{grv}$, and $h_{spr}$ using alongshore and cross-shore profiles. The geometric shape of the SAG formations was variable, but in general an absolute value of a cosine function well-represented the planform alongshore geometry and a skewed Gaussian function well-represented the shore-normal profile shape. Adopting a coordinate system of $x$ being positive offshore from the coast, and $y$ being alongshore, the functional form of the idealized depth $h(x,y)$ is given by:

$$h(x, y) = h_{base} - h_{spr}h_xh_y + \eta_{tide},$$

where $h_{base}(x)$ is the cross-shore reef form with reef flat and fore reef, $\eta_{tide}$ is the tidal level, and the cross-shore SAG variability $h_x(x)$ and alongshore SAG variability $h_y(y)$ are given by

$$h_x = \exp \left[-\frac{(x - \mu)^2}{2\sigma^2}\right],$$

$$h_y = \max \left[(1 - \alpha) \cos \left(\frac{\pi y}{\lambda_{SAG}}\right) - \alpha, 0\right],$$

where $\alpha$ is a constant.
where $\mu$ is the $x$ location of peak SAG height, $\varepsilon$ is a spreading parameter with $\varepsilon = \varepsilon_1$ for $x > \mu$ and $\varepsilon = \varepsilon_2$ for $x < \mu$ to create the skewed Gaussian form, and $\alpha$ is a coefficient depending on $W_{grv}$ and $\lambda_{SAG}$ given by:

$$
\alpha = \left| \cos \left( \frac{\pi}{2} \left( 1 + \frac{W_{grv}}{\lambda_{SAG}} \right) \right) \right| \left| 1 - \cos \left( \frac{\pi}{2} \left( 1 + \frac{W_{grv}}{\lambda_{SAG}} \right) \right) \right|.
$$

(18)

These equations were used with the typical SAG parameters of: $\lambda_{SAG} = 50$ m, $h_{spr} = 2.9$ m, $\mu = 550$ m, $\varepsilon_1 = 77$ m, $\varepsilon_2 = 20$ m, $W_{grv} = 3$ m, $\eta_{tide} = 0$, (Figure 4). Maximum depth was limited to 22 m based on $kh$ model constraints. Qualitatively, this form is similar to SAG formations in Figure 2, thus giving some confidence that this idealized model bathymetry is representative of SAG formations.

3.2 Model Parameters and Processing

Bottom roughness for the reef was evaluated using the methods of Lowe et al. [2009], assuming an average coral size of 14 cm, and thus a drag coefficient $C_d = 0.06$. Similar values of drag coefficients for coral reefs are reported in Rosman and Hench [2011]. The base-configuration model had a spatially uniform $C_d = 0.06$, with no $C_d$ variation between spurs and grooves. As grooves often do not have coral but are instead filled with sediment (see Figure 1, and Storlazzi et al., 2003), some additional runs were conducted with a spatially variable $C_d$ that was lower ($C_d = 0.01$) in the grooves to determine the potential effect of variable bottom roughness (Section 4.4). The $C_d = 0.01$ used for the sand channels was assumed to have higher roughness than for flat sand due to likely sand waves and coral debris.
Typical wind and wave conditions on Moloka‘i have been summarized in Storlazzi et al. [2011]. In general, wind speed varies from 0 to 20 m/s, and direction is variable depending on the season. Average incident wave conditions are also variable and dependent on the season, but in general from offshore buoy data the average deep-water wave height varies from 0.5 to 1.5 m, average deep-water wave period varies from 6 to 14 s, and average observed deep-water wave angle varies from 0 to 80° (0° corresponds to normally incident waves). The wave angle was assumed to follow Snell’s law in propagating from deep-water offshore to the model wave maker at 22 m depth, thus limiting the range of possible \( \theta_i \). Tidal variation for southern Moloka‘i is 0.4 m to 1.0 m.

A grid size of \( \Delta x = \Delta y = 1 \) m was used with bathymetry, as shown in Figure 4. Sponge layers were located at 60 m and 800 m offshore (Figure 4a). The wave maker center was located at 752 m (Figure 4a), with forcing incident wave height \( H_i \), period \( T_i \) and angle \( \theta_i \). The computational time step was 0.01 s, and instantaneous values of \( u, v, \eta \), and \( \nu_{br} \) were output at 0.2 s intervals. The maximum value of \( kh \) in the model domain was 1.1 for the base-configuration (offshore) and 1.5 for all runs, and is within the limits suggested by Nwogu [1993]. A biharmonic eddy viscosity \( \nu_{bi} \) of 0.2 m\(^4\)s\(^{-1}\) was used, with wave breaking parameters of: \( \delta_b = 1.2, \eta_i^{(F)} = 0.65 \sqrt{gh}, \eta_i^{(F)} = 0.15 \sqrt{gh}, \) and \( T^* = 5 \sqrt{h/g} \) as defined by Kennedy et al. [2000]. Surface forcing due to wind was input to the model assuming a typical drag law in the momentum equation,

\[
F_s = \tau_w/(h\rho) = \frac{C_{dw}U_{10}}{h\rho} |U_{10}| \rho_a,
\]

(19)
where drag $C_{dw} = 0.0015$, density of air $\rho_a = 1$ kg m$^{-3}$, and the wind velocity $U_{10}(U_{10}, V_{10})$ at a reference level of 10 m.

The model was first run in a base-configuration with model parameters typical of average conditions on Moloka‘i (Table 1) to diagnose the SAG-induced circulation. Subsequently the model parameters were varied (denoted variation models – Table 1). The variation models configuration is similar to that described previously. However, for $\theta_i$ variation, the alongshore length was extended to 700 m to allow the oblique waves to fit into the alongshore domain with periodic boundary conditions. Additionally, for $\beta_i$ variation the cross-shore dimension was adjusted so that the wave maker and sponge layers were the same distance from the SAG formations. For example, for $\beta_i = 2\%$, the cross-shore domain length was 1692 m, the wave maker was located at $x = 1466$ m, and the sponge at $x = 1512$ m. For variation in $T_i$, the cross-shore width of the wave maker was held constant at approximately 60 m. For variation in $\lambda_{SAG}$, the alongshore model length was adjusted to model $2\lambda_{SAG}$.

Model run time was 3600 s, with 3240 s of model spin-up and the last 360 s for post processing analysis. At the model spin-up time, the mean Eulerian currents at all locations in the model domain had equilibrated. Simulations conducted with variable alongshore domains that are multiples of $\lambda_{SAG}$ gave identical results, thus an alongshore domain that spanned $2\lambda_{SAG}$ was used here.
4. Results

4.1 Base-Configuration Model Results

This section describes the idealized base-configuration model based on typical parameters for southern Moloka‘i, Hawai‘i (Table 1). Results are shown for the model domain from the edge of the onshore sponge layer \(x = 60\) m to the onshore side of the wave maker \(x = 720\) m. The cross-shore variation of \(\eta\) at the end of the model run \((t = 3600\) s\()), \(H, \theta,\) and \(\bar{\eta}\), for both the spur and groove profiles are shown in Figure 5. As the waves approach the fore reef they steepen and increase in height from 1.0 m to 1.8 m (trough-to-crest) (Figure 5a), and from 1.0 m to 1.3 m (based on surface variance \(H\)) (Figure 5b). Within the surf zone (demarked by the vertical dotted lines), the waves were actively breaking, reducing \(H\) (Figure 5b). \(H\) continues to decay with onshore propagation along the reef flat. \(H\) is slightly higher along the spur, due to the effects of diffraction and refraction. The alongshore mean \(\theta\) is nearly zero along the model domain, but the alongshore maximum and minimum \(\theta\) show small oscillations induced along the reef flat due to effects of diffraction and refraction (Figure 5c). \(\bar{\eta}\) is slightly set down just before wave breaking, is set-up through the surf zone, and is fairly constant on the reef flat (Figure 5d).

This cross-shore reef setup profile is qualitatively in agreement with field observations [e.g., Taebi et al., 2011; Monismith, 2007]. There are very small \(O(1\%)\) differences in \(\bar{\eta}\) between the spur and groove profiles which are much smaller than the cross shore variability in \(\bar{\eta}\) (i.e. \(|\partial \bar{\eta}/\partial y| \ll |\partial \bar{\eta}/\partial x|\)).

The cross-shore variation of \(U_S, U_E,\) and \(U_L\) for both spur and groove profiles are shown in Figure 6. Positive velocities are oriented offshore and negative velocities are oriented onshore. \(U_S\) (computed from Eq. 7) increases from offshore to wave breaking, and decreases within the
surf zone and on the reef flat. Along the SAG system, there is a small $O(20\%)$ difference in $U_S$ between the spur and groove profiles. Model derived $U_S$ (Eq. 7) and $U_S$ based on second order wave theory (i.e. a nonlinear quantity accurate to second order in $ak$, whose origins are based in linear wave theory, Eq. A1) are similar in the shoaling fore reef region (Appendix A). Along the majority of the fore reef ($350 < x < 520$ m), $U_E$ is $O(50\%)$ larger over the groove than over a spur (Figure 6b). The circulation $U_c$ is nearly identical to $U_L$ in Figure 6(c), due to weak alongshore currents along the spur and groove profiles in this model. The predominant two-dimensional $U_L$ circulation pattern is onshore flow over the spur and offshore flow over the groove along the majority of the SAG formation up to the surf zone ($330$ m $< x < 520$ m) (Figure 7). Near the offshore end of the spur ($x \approx 550$ m), this $U_L$ circulation pattern is reversed, see Section 5.3 for further discussion on potential three-dimensional effects.

From offshore, the magnitude of $\tau_{bx}$ generally increases up to wave breaking, and slowly decreases on the reef flat Figure 6(d). Along the majority of the SAG formation up to the surf zone ($330$ m $< x < 520$ m), $\tau_{bx}$ is stronger on the spur than the groove and is oriented onshore on the spur, while oscillating sign on the groove.

### 4.2 Mechanism for Circulation

Outside the surf zone, assuming normally-incident waves, steady-state mean velocities, small alongshore currents, and no surface forcing, the phase-averaged cross-shore ($x$) momentum equation (Eq. 13) is given by

$$U_E \frac{\partial U_E}{\partial x} + u' \frac{\partial u'}{\partial x} = -g \frac{\partial \bar{h}}{\partial x} - \frac{\tau_{bx}}{\rho(h + \bar{\eta})},$$

(20)
where the terms are referenced from left to right as nonlinear advective mean (NLM), nonlinear advective wave (NLW), pressure gradient (PG), and bottom stress (BT). The remaining terms in Eq. 13 are negligible (confirmed through model results). The NLW term is analogous to the radiation stress gradient in wave-averaged models [Longuet-Higgins and Stewart, 1964] (see Appendix A for comparison of the direct radiation stress estimates with those of second-order wave theory).

The fore reef (400 m < x < 600 m) has a classic set-down balance [e.g., Bowen, 1969; Kumar et al., 2011] between PG and NLW terms (Figure 8a). Closer to where wave-breaking occurs (330 m < x < 400 m), BT also becomes important (Figure 8a). Within the surf zone (270 m < x < 330 m) and on the reef flat (x < 270 m), the classic surf zone setup (PG-NLW-F_br) and reef-flat (PG-BT) cross-shore momentum balances were obtained from the model [e.g., Monismith, 2007].

On the SAG formations, (400 m < x < 600 m), the alongshore variation of the cross-shore momentum balance (Eq. 20) shows that the PG and NLW terms do not balance and their difference is largely balanced by BT (Figure 9a). The NLM term is very small. The PG and NLW mismatch depends upon alongshore position on the SAG bathymetry (Figure 9c). The alongshore variation in NLW is primarily due to the local cross-shore slope, while the alongshore variation in PG is primarily due to the local depth (see Appendix B). On the spurs, the PG and NLW terms are basically in balance as in a classic set-down balance [Bowen, 1969], whereas on the grooves, they are out of balance, and the PG and NLW mismatch is balanced by the BT. The residual forcing accelerates the flow until BT is large enough to balance it which drives an offshore $U_E$. $U_S$ is very weakly alongshore variable so the alongshore variation in $U_L$. 

JGR-Oceans – Ref 2012JC008537 Rev 2, 4/24/2013 16
and hence the circulation, is largely due to $U_E$ (Figure 9b). Note that the fore reef circulation does not depend on wave breaking within the surf zone (confirmed through separate model runs with smaller $H_i$ that did not have a surf zone).

### 4.3 Effects of Hydrodynamic Conditions and SAG Geometry

The base-configuration parameters were varied in the model (denoted variation models, Table 1) to assess their effect on $U_c$ and $\overline{v_{bx}}$ at a reference location ($x_r = 440$ m, $y_r =$ spur top) as a representative location to assess the hydrodynamics. This location captures the main cross-shore $U_L$ circulation cell for a wide range of modeled parameters (e.g., $H_i$, $C_d$, $h_{spr}$, etc.). To evaluate relative changes to $U_c$ and $\overline{v_{bx}}$, these are normalized by the base-configuration values at the reference location:

$$\overline{U_c} = \frac{U_c}{U_{cb}},$$
$$\overline{\overline{v_{bx}}} = \frac{v_{bx}}{\overline{v_{bx}}},$$

with $U_{cb}(x_r,y_r) = -0.0060$ m/s and $\overline{v_{bx}}(x_r,y_r) = -0.37$ Pa, representing the base-configuration. The reference water depth $h_r$ is the depth at the reference location $h(x_r,y_r)$. For the variation in slope $\beta_i$ and cross-shore location $\mu$ models, the cross-shore reference location $x_r$ was positioned in the same relative cross-shore position on the SAG formation for each geometric configuration (i.e. base-configuration $\mu = 500$ m and $x_r = 440$ m; for $\mu = 550$ m, $x_r = 490$ m).

The modeled dependence of $\overline{U_c}$ and $\overline{\overline{v_{bx}}}$ on the model variables are shown in Figure 10 and Figure 11, respectively. From a maximum at a spur-normal wave incidence angle ($\theta_i = 0^\circ$), $\overline{U_c}$ quickly decreases to nearly zero with oblique incidence ($\theta_i = 20^\circ$), with $\overline{\overline{v_{bx}}}$ remaining nearly constant (Figure 10a and Figure 11a). $\overline{U_c}$ and $\overline{\overline{v_{bx}}}$ increase linearly and quadratically, respectively with increasing $H_i$ (Figure 10b and Figure 11b). Increased $T_i$ slightly decreases $\overline{U_c}$.
but shows oscillations in $\tau_{bx}$ (Figure 10c and Figure 11c). The effects of refraction/diffraction are likely important here. $\overline{u_c}$ and $\tau_{bx}$ weakly decrease with increasing $\eta_{tide}$, (Figure 10d and Figure 11d). $\overline{u_c}$ and $\tau_{bx}$ show no variation with $U_{10}$ as expected due to closed cross-shore boundaries (Figure 10e and Figure 11e). Here, wind and waves are not coupled, so increased wind forcing does not influence wave growth. Increased $V_{10}$ decreases $\overline{u_c}$ (Figure 10f). The circulation cells driven by the SAG bathymetry (Figure 7) are essentially overwhelmed by the increasingly stronger alongshore current, which decreases $\overline{u_c}$ proportional to $\cos(\varphi)$ (Eq. 10). This is similar to oblique wave incidence. Increased $V_{10}$ shows a slight decrease in $\tau_{bx}$ (Figure 11f).

$\overline{u_c}$ and $\tau_{bx}$ vary inversely with decreasing $h_{spr}$, (Figure 10g and Figure 11g). Similarly, $\overline{u_c}$ and $\tau_{bx}$ vary inversely with increasing spur cross-shore position $\mu$ (Figure 10h and Figure 11h). The dependence of $\overline{u_c}$ on $\lambda_{SAG}$ shows small peaks at 80 m and 200m, while $\tau_{bx}$ shows a broad, but weak peak centered around 200 m (Figure 10i and Figure 11i). The hydrodynamics of different $\lambda_{SAG}$ will be discussed in more detail in Section 5.2. Increased reef $C_d$ shows decreased $\overline{u_c}$ and increased $\tau_{bx}$, (Figure 10j and Figure 11j). Increased $W_{grv}$ to $\lambda_{SAG}$ ratio, shows increased $\overline{u_c}$ and nearly constant $\tau_{bx}$ (Figure 10k and Figure 11k). $\overline{u_c}$ and $\tau_{bx}$ linearly increase with $\beta_l$ (Figure 10l and Figure 11l).

The effect of particular model variables (Table 1) on SAG-influenced $U_E$ and $\tau_{bx}$ on the fore reef can be derived from a simplified scaling (Appendix B) of the dominant cross-shore $x$ momentum balance (Section 4.2).
\[
U_E \approx \frac{\pi \sqrt{g \beta_{AS} H_{AS}}}{16 \nu d \sqrt{h_{AS}}} \left[ \frac{Y_u^{1/2}}{Y_h^{1/2}} - \frac{Y_h^{3/2}}{Y_u^{1/2}} \right],
\]
(23)
\[
\tilde{\tau}_{bx} \approx \frac{\rho g \beta_{AS} H_{AS}^2}{16 h_{AS}} \left[ \frac{Y_u^{1/2}}{Y_h} - \frac{Y_h}{Y_{u_{h}}^{1/2}} \right],
\]
(24)

which are Eq. B9 and Eq. B11 in Appendix B, respectively. \(_{AS}\) and \(_'\) denote an alongshore average and alongshore deviation respectively. The local depth factor \(\gamma_h = 1 + h'/h_{AS}\), \(\gamma_b\) and \(\gamma_H\) are similarly defined. \(\gamma_h\) and \(\gamma_u\) are \(kh\) dependent correction terms defined in Appendix B.

The terms in brackets in Eq. 23 and 24 contain the local alongshore variability in \(U_E\) and \(\tilde{\tau}_{bx}\); the dominant factors are local depth (\(\gamma_h\)) and local slope (\(\gamma_b\)). Since the strength of \(U_c\) is due to alongshore variations in \(U_E\), the nondimensional scaled \(U_c\) to first order scales proportionally to \(U_E/\bar{U}_{E_b}\), where \(\bar{U}_{E_b}\) is the base condition \(U_E\). Thus, for terms that vary independently in Eq. 23 and 24, with normally-incident waves on the spur, and relatively small \(h_{sp}\) (\(\beta_{AS} \approx \beta_f\)):

\[
\tilde{U}_c \propto \tilde{U}_c \left[ \frac{H_i}{h_{ib}}, \frac{\sqrt{h_b}}{\sqrt{h}}, \frac{C_{db}}{C_d}, \frac{\beta_f}{\beta_{fb}} \right],
\]
(25)
\[
\tilde{\tau}_{bx} \propto \tilde{\tau}_{bx} \left[ \frac{H_i^2}{H_{ib}^2}, \frac{h_b}{h}, \frac{\beta_f}{\beta_{fb}} \right],
\]
(26)

where \(H_{ib}, h_b, C_{db}, \) and \(\beta_{fb}\) are the base condition \(H_i, h, C_d,\) and \(\beta_f\) respectively. Eq. 25 and 26 capture the first order effects of variables on \(\tilde{U}_c\) and \(\tilde{\tau}_{bx}\), but do not capture more complex processes such as wave refraction/diffraction, local alongshore-variability of \(h, H,\) and \(\beta\) (Eq. 23 & 24), as well as other second order effects ignored in this scaling (such as correlations between \(\eta\) and \(u'\) in the BT term). The results for \(\tilde{U}_c\) and \(\tilde{\tau}_{bx}\) based on the model (Eq. 21 & 22) and
scaling approximation (Eq. 25 & 26) are generally similar [Figure 10 (b, d, h, j, l) and Figure 11 (b, d, h, l)], with differences likely due to these more complex processes.

4.4 Effect of Spatially Variable Drag Coefficient

The base-configuration model had spatially uniform drag coefficient $C_d$. However, on typical SAG formations, spurs are covered with hydraulically rough corals (high $C_d$), while the grooves are often filled with less-rough sediment (low $C_d$) (example Figure 1). The difference in $C_d$ between spur and groove could also have consequences on the net circulation, independent of SAG geometry. To test this idea, a separate model run was performed with SAG formations ($h_{spr} = 2.9$ m), but with spatially variable $C_d$ between spurs ($C_d=0.06$) and grooves ($C_d=0.01$). A Lagrangian circulation pattern similar to the base-configuration (Figure 7) was created, but of slightly larger magnitude ($\approx 4\%$). In another model run assuming no SAG formations ($h_{spr} = 0$ m) but with spatially variable $C_d$ between spurs ($C_d=0.06$) and groove ($C_d=0.01$), a Lagrangian circulation pattern similar to the base-configuration (Figure 7) was created, but much smaller, $O(10\%)$. Thus, SAG bathymetry is the primary driver of the Lagrangian circulation patterns shown in Figure 7, while alongshore differences in $C_d$ between the coral and sediment-filled grooves have a negligible role. Reef-scale $C_d$, however, is important to the overall circulation as it sets the magnitude of the circulation (Section 4.3).

5. Discussion

5.1 Relative Effect of Return Flow to SAG-Induced Circulation

Many reefs have channels or lagoons onshore of the reef flat with a connection back to the open ocean (open back reef), while other reefs have a closed back reef with no ocean
connection [Spalding et al., 2001]. SAG formations are often found on the fore reefs for both open and closed back reef geometries. A net onshore flow over reef flats has been measured in numerous field experiments on reefs with such open ocean back connections [Symonds et al., 1995; Bonneton et al., 2007; Monismith, 2007]. The funwaveC model has a closed onshore boundary at $x = 0$, which is reasonable for the closed back reef on southern Moloka‘i, Hawai‘i. A relevant question then is for open back reefs, how strong is the SAG-driven circulation on the fore reef compared to the net onshore flow driven by the open ocean connection?

On the reef flat, neglecting bottom boundary layer wave dissipation, the primary momentum balance on the reef flat is between pressure gradient and bottom stress [e.g., Hearn, 1999],

$$ g(h + \bar{\eta})\nabla \bar{\eta} = -\frac{C_d}{(h + \bar{\eta})^2} q_E |q_E|, $$

(25)

where $q_E = (h + \bar{\eta})U_E$ is the mean Eulerian transport. If the overall reef flat depth change is assumed to be small, the reef flat flow can be approximated by,

$$ q_E \approx (gh/C_d)^{1/2} \left(\frac{\bar{\eta}_r - \bar{\eta}_L}{L_r}\right)^{1/2}, $$

(26)

where $h$ is the mean depth on the reef, $\bar{\eta}_r$ is the setup at the end of breaking, $\bar{\eta}_L$ is the mean surface at the lagoon, and $L_r$ is the length of the reef flat. For the modeled base-configurations, at the end of model domain where all wave energy is dissipated ($x = 0$ m), the setup $\bar{\eta}_r$ is 0.025 m (not shown in Figure 5), while the offshore ($x = 720$ m) setup $\bar{\eta}_L$ is -0.027 m (Figure 5). Using $C_d = 0.06$, varying $L_r$ from 100 m to 2000 m, and varying $h$ from 0.5 m to 1.5 m, the results indicate that the $q_E$ has the potential to vary from -0.02 to -0.5 m$^2$s$^{-1}$ (directed onshore). The funwaveC
model results indicate that the SAG formation-induced mean Lagrangian transport \( q_L = (h + \bar{\eta})U_L \) is \(-0.06 \text{ m}^2 \text{ s}^{-1}\) and \(0.09 \text{ m}^2 \text{ s}^{-1}\) on the spur and groove, respectively (Figure 6c).

Although this analysis is qualitative, it indicates that it is possible for transport induced over the reef flat to be of a similar magnitude as the SAG-induced circulation. Under certain conditions, such as strong offshore wave forcing inducing strong transport over the reef flat, the onshore transport on the spur would be strengthened, while the offshore transport on the grooves would be reduced, or potentially reversed. If there is no reef pass or back channel, i.e., a pure fringing reef like at Moloka‘i, the SAG induced circulation will likely be the only fore reef exchange. In all cases, at shallow depths the net Lagrangian flow over the spurs is onshore.

5.2 SAG Wavelength

Waves encountering SAG formations is analogous to the classical problem of waves propagating through a breakwater gap [Penney and Price, 1952]. In the latter case, for a breakwater gap less than one wavelength, the waves in the lee of the breakwater propagate approximately as if from a point source; diffraction is predominant within several wavelengths of the gap and further away, refraction dominated [Penney and Price, 1952; Dean and Dalrymple, 1991]. Although SAG formations are submerged (instead of protruding from the surface), and their alongshore shape is rounded (instead of vertical), wave transformation over SAG formations may have some qualitative similarity to the breakwater gap where \(\lambda_{SAG}/2\) corresponds to an approximate gap scale. Thus for \(\lambda_{SAG}\) much less than the surface gravity wavelength, the wave transformation over the SAG formations may be dominated by diffraction, which tends to alongshore “diffuse” wave height, whereas for \(\lambda_{SAG}\) much larger than the surface gravity wavelength, refraction dominates the wave transformation. For wavelength larger than the gap
scale, the effect of refraction becomes important several wavelengths from the end of the spur or approximately 400 m as shown in the oscillations in $\theta_m$ for $x < 350$ m (Figure 5c).

For the base-configuration, the surface gravity wave wavelength over the SAG formations varied from 115 m (near the front face) to 50 m (near the surf zone). For small $\lambda_{SAG} < 100$ m, the fore reef $H$ difference between the spur and groove is small and grows slowly with $\lambda_{SAG}$ (Figure 12a) and $\theta$ is zero at both spurs and grooves (Figure 12b) consistent with diffraction being dominant. At larger $\lambda_{SAG}$ (>100 m) the spur-groove difference in $H$ grows rapidly and equilibrates at $\lambda_{SAG} > 200$ m (Figure 12a). Similarly, the alongshore maximum and minimum $\theta$ increases similar to the wave height equilibrating to $5^\circ$ at $\lambda_{SAG} > 200$ m (Figure 12b). This large $\lambda_{SAG}$ behavior is consistent with refraction being dominant.

The effect of $\lambda_{SAG}$ variation on the SAG circulation is seen through the $x$-momentum terms on the groove which has the largest signal (Figure 13a). The PG and NLW mismatch balanced by BT (Section 4.2) increases with $\lambda_{SAG}$ driving an offshore $U_E$ that also generally increases with $\lambda_{SAG}$ (Figure 13b). $U_S$ generally increases with $\lambda_{SAG}$, but has opposite sign of $U_E$, resulting in $U_L$ (and thus $U_c$) that has a maximum near $\lambda_{SAG} = 80$ m, with a secondary maximum at larger $\lambda_{SAG}$ (Figure 13b). $\tau_{bx}$ (Figure 13c) is a function of $H^2$ (Section 4.3). Thus for $\lambda_{SAG}$ less than 90 m, $\tau_{bx}$ remains relatively constant, while for larger $\lambda_{SAG}$, $\tau_{bx}$ increases to a local maxima at $\lambda_{SAG}$ equal to 200 m due to the effects of diffraction/refraction (Figure 13c). It appears the maximum circulation and bottom stress occurs when the SAG wavelength allows for the effects of diffraction to create alongshore differences in wave height without changing the mean wave angle, in which case the SAG formations are most efficient at driving Lagrangian circulation cells.
5.3 Two-Dimensional SAG Circulation and Potential Three-Dimensional Effects

The predominant two-dimensional horizontal Lagrangian circulation pattern induced by the waves is counter-rotating circulation cells. From $x \approx 530$ m to the surf zone, transport is onshore over the spur and offshore over the groove; while from the end of the spur to $x \approx 530$ m the flow direction is reversed (Figure 7). A wide range of hydrodynamic conditions and SAG geometries were modeled (Section 4.3). For all modeled conditions except strongly angled waves (high $\theta$) or strong alongshore currents, the waves over SAG formations induce the same basic Lagrangian circulation cells.

The present study focuses on the barotropic (depth-averaged) circulation. The modeled conditions are within the range of values of $kh$ for which the governing equations [Nwogu, 1993] and associated numerical methods [Feddersen, 2007] are valid. Thus the depth-averaged flows in this study should be accurate. Even so, it is reasonable to expect three-dimensional flow effects to become important, especially in the deeper areas of the SAG model domain. For example, while the vertical structure of $U_s$ is easily calculated, *funwave C* only calculates depth-averaged mean Eulerian flows ($U_E$) meaning that the vertical structure of the mean Lagrangian flows remain to be determined. Additionally, the model does not represent more complicated three-dimensional flow processes such as separation that might occur. Clearly, these more complicated flow features could have important hydrodynamic and biological implications. Thus, further study of the wave-induced currents over the SAG geometry using fully three-dimensional modeling techniques or field studies would seem warranted.
6. Conclusions

In summary, a time-dependent Boussinesq wave model, funwaveC that resolves individual waves and parameterizes wave breaking was used to numerically simulate current velocities and sea surface height along SAG formations based on idealized bathymetry from Moloka‘i, Hawai‘i. The predominant two-dimensional Lagrangian circulation pattern is counter-rotating circulation cells induced by the shoaling wave field over the SAG bathymetry. In shallow depths, transport is directed onshore over the spur and offshore over the groove, while near the end of the spur in deeper water the circulation is reversed. The primary driver of these Lagrangian circulation patterns is the waves interacting with the SAG bathymetry, not alongshore differences in bottom drag due to variation in a drag coefficient. The dominant phase-averaged momentum balance is between pressure gradient and nonlinear wave advection terms on the fore reef. The alongshore variation of the x-momentum terms shows that the pressure gradient and nonlinear wave advection term are not in exact balance and their difference is balanced by bottom stress.

The effect of model variables on circulation $U_c$ and cross-shore average bottom stress, $\tau_{bx}$ on the fore reef was approximated using scaling arguments of the dominant cross-shore $x$ momentum balance. The model results show $U_c$ varies approximately proportionally to $H_i$, $h^{-1/2}$, $C_d$, and $\beta_f$, consistent with a simple scaling. The parameters that created the strongest $U_c$ were spur-normal incident waves ($\theta_i = 0^\circ$), increased $H_i$, no alongshore currents, and increased $h_{spr}$.

The present study focuses on the barotropic (depth-averaged) circulation, but it is reasonable to expect three-dimensional flow effects to become important, especially in the deeper areas of the SAG model domain. Thus, further study of the wave-induced currents over
the SAG geometry using fully three-dimensional modeling techniques or field studies are a logical next step.

Many reefs have channels or lagoons onshore of the reef flat with a connection back to the open ocean, while other reefs have a closed back reef with no connection [Spalding et al., 2001]. Using an order of magnitude analysis, results indicate that it is possible for flow induced over the reef flat to be of a similar magnitude as the circulation induced by the SAG formations. Under all onshore reef flows at shallow depths, the net Lagrangian flow over the spurs remains directed onshore.

An investigation was conducted into the hydrodynamic behavior of SAG formations of different SAG wavelength. It appears the maximum circulation and low bottom stress occurs when the SAG wavelength allows for the effects of diffraction to create alongshore differences in wave height without changing the mean wave angle, thus the SAG formations are most efficient at driving Lagrangian circulation cells.

The typical circulation pattern noted in this study likely brings low-sediment, high “food” water from the open ocean up over the corals on the spur; while simultaneously transporting coral debris and sediment from the surf zone and reef flat along the groove sand channels and away from the reef, (assuming low alongshore exchange between spurs and grooves). Average cross-shore bottom shear stress is stronger on the spur than the groove, thus for large wave events that generate shear stress above the capacity of the corals, the corals on the spur would exceed their capacity and break. However, the increased bottom stress on the spur also likely allows for sediment shedding towards the grooves and possibly more nutrient exchange due to increased turbulence on the spur under certain conditions. Thus, while the net effect of bottom
shear stress on coral growth remains unclear, increased circulation may favor growth on the coral spur and inhibit coral growth in the groove. Based on variations in the assumed model parameters, some of the strongest factors affecting SAG circulation include spur-normal waves ($\theta=0^\circ$), increased wave height, and increased spur height. If increased circulation is favorable to coral growth, the modeling results are qualitatively consistent with field observations that SAG formations are orthogonal to typical predominant incident wave angle and are largest and most well developed in areas of larger incident waves [Munk and Sargent, 1954; Roberts, 1974; Storlazzi et al., 2003].

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Appendix A – Comparison to Second Order Wave Theory

It is of practical interest to compare the Stokes drift and radiation stress obtained from second order wave theory (i.e. nonlinear quantities accurate to second order in $ak$, whose origins are in linear wave theory) to the results obtained from the nonlinear Boussinesq model. The Stokes drift can be approximated from the wave height, assuming second order wave theory by [e.g., Svendsen, 2007]: 
\[ U_s = \frac{gkh^2}{8h\sigma} \cos \theta, \]  
(A1)

where \( U_s \) is in the \( x \) direction, \( H \) is the wave height, \( k \) is the wavenumber, and \( \sigma \) is the wave radian frequency which are related in the usual dispersion relation \( (\sigma^2 = gk \tanh kh) \). The results for \( U_s \) using \( H \) (Eq. A1) and the difference in Eulerian and Lagrangian velocities (Eq. 7) methods have fairly good agreement offshore of wave breaking \( (x > 350 \text{ m}) \), where the waves are weakly nonlinear (Figure A1a). Agreement is poor in the surf zone and reef flat due to strong nonlinearity in the wavefield (Figure 5a).

The radiation stress tensor \( S_{ij} \) correct to \( O(H/L)^3 \) is given by [e.g. Mei et al, 2005]:

\[
S_{ij} = \rho \int_{-h}^{h} \mathbf{u}_i \mathbf{u}_j dz + \delta_{ij} \left\{ \frac{\rho g}{2} \tilde{\eta}^2 - \rho \int_{-h}^{h} \tilde{w}^2 dz \right\},
\]  
(A2)

where \( L \) is the wavelength \( (k/2\pi) \) and \( \tilde{u} \) is based on \( \tilde{u} = u - U_L \). For the nonlinear model dynamics, \( S_{ij} \) can be approximated by assuming linear wave theory for the third term for vertical velocity \( \tilde{w} \) above, and using the instantaneous depth averaged velocities, which can account for weak reflections:

\[
S_{ij} = \rho (\tilde{\eta} + h) \tilde{u}_i \tilde{u}_j + \delta_{ij} \frac{\rho g}{2} \tilde{\eta}^2 \left( \frac{2kh}{\sinh 2kh} \right),
\]  
(A3)

Eq. A3 can be evaluated from the model results assuming the frequency is known. For progressive waves \( S_{11} \) \( (S_{xx}) \) is given by [Longuet-Higgins and Stewart, 1964]:

\[
S_{11} = S_{xx} = \frac{E}{2} \left[ \frac{2C_g}{C} \cos^2 \theta + \left( \frac{2C_g}{C} - 1 \right) \right],
\]  
(A4)

where \( E \) is the energy, \( C \) is the phase velocity, \( C_g \) is the group velocity, and \( \theta \) is the wave angle. While the magnitude of \( S_{xx} \) from the two methods is similar (Figure A1b), the results from linear
wave theory show more local variability in the cross shore gradient of $S_{xx}$ (i.e. $\partial S_{xx}/\partial x$)
which is the result of small cross-shore oscillations in $H$ (Figure 5b).

Appendix B – Scaling of the Boussinesq Equation

Here we present an approximate scaling for the SAG circulation developed to help
explain the results in Section 4.3. The circulation is given by $U_c = (U_L + U_S) \cos(\varphi)$ (Eq. 10).
The alongshore variation in $U_L$ (and thus $U_c$) is primarily a result of alongshore variation in $U_E$
not $U$, (which is nearly alongshore uniform) (Section 4.2). Thus, to first order, the strength of $U_c$
is due to alongshore variations in $U_E$.

On the fore reef, away from the surf zone ($\eta < h$), but not too deep ($kh < 1.5$), the
primary phase-averaged cross-shore ($x$) momentum balance (Eq. 20) is between NLW, PG and
BT (Figure 8a).

$$u' \frac{\partial u'}{\partial x} \approx -g \frac{\partial \bar{h}}{\partial x} \frac{\tau_{bx}}{\rho(h + \bar{\eta})}.$$  \hspace{1cm} (B1)

Linear wave theory is assumed for normally incident ($\theta = 0$) waves of the form $\eta = (H/2) \cos(\omega t)$, with wave speed $C$ can be expressed as $C = \omega / k = \sqrt{\gamma_c gh}$, where employing
the dispersion relation $\omega^2 = gk \tanh kh$, a correction to the shallow water wave speed is
$\gamma_c = \tanh(kh)/(kh)$. Taking the standard form for linear wave velocity $u'$ [e.g. Dean and
Dalrymple, 1991], is evaluated at $z_r = -ah$, with $a = 0.531$ [Nwogu, 1993]. The wave velocity
is then

$$u' = U_0 \cos(\omega t),$$  \hspace{1cm} (B2)

where $U_0 = (\sqrt{g\gamma_a H})/(2 \sqrt{h})$, and the $kh$ dependent wave velocity terms are combined
\[ \gamma_u = \cosh^2[(1 - \alpha)kh] \frac{2kh}{\sinh(kh)} \]  \hspace{1cm} (B3)

For small \( kh \), \( \gamma_u = 1 \), for \( kh = 1 \), \( \gamma_u = 0.68 \). The NLW wave term from Eq. B1 can then be evaluated using Eq. B2,

\[ u' \frac{\partial u'}{\partial x} = \frac{g \gamma_u H}{16h^2} \left[ 2h \frac{\partial H}{\partial x} - H \frac{\partial h}{\partial x} \right], \hspace{1cm} (B4) \]

where \( \partial \gamma_u / \partial x \) is very small \([O(2kh\omega\beta / \sqrt{gh}) \sim O(10^{-3} \text{ m}^{-1})]\) confirmed through model results and first-order scaling. The mean set-down for alongshore uniform bathymetry in a classic pressure gradient – radiation stress balance offshore of the surf zone is given by \( \bar{\eta} = -\left( kH^2 \right) / \left[ 8 \sinh(2kh) \right] \) \cite{Longuett-Higgins_1962, Bowen_1969}. This solution is based on alongshore uniform bathymetry, therefore it is most appropriate that \( k \) and \( h \) are taken as the alongshore average denoted by \( k_{AS} \) and \( h_{AS} \) respectively. The set-down can then be written as,

\[ \bar{\eta} = -\frac{\gamma_s H^2}{16h_{AS}}, \hspace{1cm} (B5) \]

where the \( kh \) dependent set-down terms are given by \( \gamma_s = (2k_{AS}h_{AS}) / \sinh(2k_{AS}h_{AS}) \). For small \( kh \), \( \gamma_s = 1 \), for \( kh = 1 \), \( \gamma_s = 0.55 \). The modeled mean set-down was well approximated by Eq. B5. The PG term from Eq. B1 is then evaluated using Eq. B5,

\[ g \frac{\partial \bar{\eta}}{\partial x} = -\frac{g \gamma_s H^2}{16h_{AS}^2} \left[ 2h_{AS} \frac{\partial H}{\partial x} - H \frac{\partial h_{AS}}{\partial x} \right], \hspace{1cm} (B6) \]
where $\partial \eta / \partial x$ is very small \($O(2k_{AS}h_{AS}\omega \beta_{AS}/\sqrt{gh_{AS}}) \sim O(10^{-3} \text{ m}^{-1})$ confirmed through model results and first-order scaling\). Let the local slope $\partial h / \partial x$ and alongshore average slope $\partial h_{AS} / \partial x$ be denoted by $\beta$ and $\beta_{AS}$ respectively (note for small $h_{spr}$, $\beta_{AS} \approx \beta_f$). The sum of NLW and PG terms (Eq. B4 and B6) can be rearranged into two terms, ignoring common terms, one with $(\partial H / \partial x)(h^{-1} - h_{AS}^{-1})$ and the second with $(H/2h)(-h^{-1}\beta + h/h_{AS}^2 \beta_{AS})$. The first is much smaller than the second (confirmed through model results) since $\partial H / \partial x$ is much smaller than $H/2h$ and differences in local vs. alongshore depths are linear in the first term, but squared in the second. The sum of NLW and PG terms becomes,

$$
\mathbf{\bar{u}} \frac{\partial \mathbf{\bar{w}}}{\partial x} + g \frac{\partial \bar{\eta}}{\partial x} \approx \frac{g\gamma_u \beta H^2}{16h^2} \left[ -1 + \left( \frac{\gamma_{\eta} \beta_{AS} h^2}{\gamma_u \beta h_{AS}^2} \right) \right]
$$

(B7)

The modeled NLW+PG was reasonably approximated by Eq. B7. For purposes of scaling, the BT term in Eq. B1 is approximated by

$$
\frac{\tau_{bx}}{\rho(h + \eta)} \approx \frac{\tau_{bx}}{\rho \bar{h}}
$$

(B8)

where it is assumed $h \gg \eta$. Combining equations B1, B7, and B8 with some rearrangement yields,

$$
\tau_{bx} \approx \frac{\rho g \gamma_u \beta H^2}{16h} \left[ 1 - \left( \frac{\gamma_{\eta} \beta_{AS} h^2}{\gamma_u \beta h_{AS}^2} \right) \right]
$$

(B9)

where the terms in the large bracket above come from the NLW and PG terms respectively. In a weak current, small angle regime, where $u' \gg U_e$ is small, for monochromatic, unidirectional waves, the mean bottom stress $\bar{\tau}_b$ is commonly parameterized by [e.g. Feddersen et al., 2000]:

JGR-Oceans – Ref 2012JC008537
Rev 2, 4/24/2013
31
\[ \bar{r}_b \approx \frac{4}{\pi} \rho C_d U_0 U_E, \]  

Combining equations B9 and B10 with some rearrangement yields,

\[ U_E \approx \frac{\pi \sqrt{g} \gamma_y \beta H}{16 C_d \sqrt{h}} \left[ 1 - \frac{\left( \gamma_y \beta_{AS} h^2 \right)}{\gamma_u \beta_{AS} h^2} \right] \]  

where the terms in the large bracket above come from the NLW and PG terms respectively.

Separating alongshore variable \( h, H \) and \( \beta \) into an alongshore average (\( _{\text{AS}} \)) and a local alongshore deviation (\( _' \)) yields, \( h = h_{\text{AS}} + h' = h_{\text{AS}} \gamma_h, H = H_{\text{AS}} + H' = H_{\text{AS}} \gamma_H, \) and \( \beta = \beta_{\text{AS}} + \beta' = \beta_{\text{AS}} \gamma_\beta, \) where the local depth, local wave height, and local slope factors are given by

\[ \gamma_h = 1 + h'/h_{\text{AS}}, \gamma_H = 1 + H'/H_{\text{AS}}, \gamma_\beta = 1 + \beta'/\beta_{\text{AS}} \text{ respectively.} \]

Substituting these expressions into Eq. B9 and Eq. B11 and rearranging so that all alongshore variability (i.e., \( \gamma_h, \gamma_H, \gamma_\beta, \gamma_u \)) is in the parentheses yields,

\[ \bar{r}_{bx} \approx \frac{\rho g \beta_{AS} h_{\text{AS}}^2}{16 h_{\text{AS}}} \left[ \frac{\gamma_u \gamma_\beta y_h^2}{\gamma_h} - \gamma_H y_H^2 \right], \]  

\[ U_E \approx \frac{\pi \sqrt{g} \beta_{AS} H_{\text{AS}}}{16 C_d \sqrt{h_{\text{AS}}}} \left[ \frac{\gamma_u^{1/2} \gamma_\beta y_h^{3/2}}{\gamma_h^{1/2}} - \gamma_H^{3/2} y_H^{1/2} \right]. \]  

The first term in brackets of Eq. B13 originates from the NLW term and is denoted NLW* (note change of sign in NLW from Figure 8a), while the second originates from the PG term and is denoted PG*. The alongshore variability in NLW* is most affected by \( \gamma_\beta \) while \( \gamma_u, \gamma_H \) and \( \gamma_h \) have a minor effect (Figure B1a). The alongshore variability in PG* is most affected by \( \gamma_h \) with little to no effect from \( \gamma_H, \gamma_h \) and \( \gamma_u \) (Figure B1b). Thus, the alongshore variability in \( U_E \) (and thus \( U_c \)) is primarily the result of a mismatch between the local slope coefficient \( \gamma_\beta \) and the local depth.
coefficient $\gamma_h$ to the 3/2 power. The alongshore variation in $U_E$ shows good agreement between model results and Eq. B13 (Figure B1c).

Equation B13 is highly approximate to $O(H/h)^2$, but explains to first order the $U_c$ dependence on model parameters, as discussed in Section 4.3. Note that if the bathymetry is alongshore-uniform ($\gamma_\eta = \cosh^2 [(1 - \alpha)kh] \gamma_u$, $\gamma_h = \gamma_H = \gamma_\beta = 1$) and Eq. B13 will predict alongshore-uniform $U_E > 0$ (directed offshore); in this case, second order effects ignored in this scaling would become important.

References


Figure Captions

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reef slope $\beta_f$. Scaling approximation only shown on (b, d, h, j, and l), blue circle indicates base-configuration.

Figure 11. Variation of model parameters and their effect on normalized average cross-shore bottom stress $\tau_{bx}^v$ for model results (red solid) and scaling approximation (Eq. 26) (black dash) at $x_r = 440$ m, $y_r = 50$ m (spur) as a function of model variables (a) incident wave angle $\theta$, (b) incident wave height $H_i$ (note larger scale), (c) incident wave period $T_i$, (d) depth as a function of tide level $\eta_{tide}$, (e) cross-shore wind $U_{10}$, and (f) alongshore wind $V_{10}$. (g) spur height $h_{spr}$, (h) depth as a function of cross-shore location $\mu$, (i) SAG wavelength $\lambda_{SAG}$, (j) drag coefficient $C_d$, (k) fraction groove width $W_{grv}/\lambda_{SAG}$, and (l) fore reef slope $\beta_f$. Scaling approximation only shown on (b, d, h, and l); blue circle indicates base-configuration.

Figure 12. Variation of wave height $H$ and wave angle $\theta$ with SAG wavelength $\lambda_{SAG}$ at $x = 440$ m. (a) alongshore mean $H$ (solid) and max/min $H$ (dash), (b) alongshore mean $\theta$ (solid) and alongshore max/min $\theta$ (dash).

Figure 13. Variation of $x$-momentum terms, velocity, circulation and average bottom shear with SAG wavelength $\lambda_{SAG}$ at $x = 440$ m. (a) phase-averaged $x$-momentum significant terms NLM, NLW + PG, and BT; residual error is small, at groove $y = 1.5\lambda_{SAG}$ (b) $U_E$, $U_S$ and $U_L$ velocities, at groove $y = 1.5\lambda_{SAG}$ and (c) normalized circulation $U_c$ (red) and normalized bottom shear stress $\tau_{bx}^v$ (black), at spur $y = \lambda_{SAG}$.

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## Tables

Table 1. Parameters used for base-configuration model, and range of parameters for variation models

<table>
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<tr>
<th>Model/Variable</th>
<th>Base-Configuration Model</th>
<th>Variation Models Min</th>
<th>Variation Models Max</th>
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<td>(\theta_i^\circ)</td>
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<td>(V_{10}) (m/s)</td>
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<tr>
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<td>(\beta_i)</td>
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</table>
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